Les Cahiers du GERAD ISSN: 0711–2440

A Differential Game of Joint Implementation of Environmental Projects

M. Breton, G. Zaccour
M. Zahaf

G–2004–10

February 2004

A Differential Game of Joint Implementation of Environmental Projects

Michèle Breton

GERAD and CREF, HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Georges Zaccour
Mehdi Zahaf

GERAD and HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

February, 2004

Les Cahiers du GERAD

G–2004–10

Abstract

This paper proposes a two-player, finite-horizon differential game model to analyze joint implementation in environmental projects, one of the flexible mechanisms considered in the Kyoto Protocol. Our results show that allowing for foreign investments increases the welfares of both parties involved in the project. Further, imposing an environmental target constraint does not necessarily deteriorate the payoffs of both players. Finally, a leakage effect does occur when foreign investments are possible.

Keywords: Environment, Joint implementation, Differential games.

Résumé

L'article propose un jeu différentiel pour analyser les stratégies d'investissement dans des projets environnementaux dans le cadre de l'application jointe, un des mécanismes flexibles préconisés dans le Protocole de Kyoto. Les résultats montrent que permettre aux joueurs d'investir l'étranger induit une amélioration de leur bien-être. Par ailleurs, l'imposition d'une contrainte environnementale ne détériore pas nécessairement les gains des joueurs. Pour terminer, l'investissement dans des projets environnementaux à l'étranger résulte par une augmentation des émissions localement.

Mots Clefs: Environnement, application jointe, jeux différentiels.

Acknowledgments: This research was supported by NSERC, Canada.

1 Introduction

In Breton et al. (2003), a game theoretic approach has been used to study joint implementation of environmental projects in a static setting. The objective of this follow-up paper is to analyze investment and emissions strategies as well as welfare implications of joint implementation in a dynamic context.

In a joint implementation endeavour, a country invests, alone or with other parties, in an environmental project located in another country and gets in return *emissions reduction units* (ERU). Such option is part of the so-called flexible mechanisms, entrenched in the Kyoto Protocol, and whose objective is to let the signatories reach their environmental targets at the lowest possible cost[1]. If the location where pollution takes place is not an issue, then the countries should be clearly better off when abating in the cheapest place.

The context we wish to deal with is the following. Consider a finite planning horizon and assume that each one of two countries faces an environmental constraint in the form of a terminal target on pollution stock. Fulfilling this constraint can be done by reducing industrial production and/or by implementing environmental projects to reduce emissions at home or abroad. The aim is then to characterize the driving forces of the joint implementation mechanism (or foreign investment) and to check if it is Pareto improving with respect to autarky, a scenario in which the players invest only locally in environmental projects.

There is a dense literature which has adopted dynamic games as a methodological framework to deal with global environmental issues, e.g., van Der Ploeg and de Zeeuw (1992), Dockner and Long (1993), Martin et al. (1993), Haurie and Zaccour (1995), Kaitala and Pohjola (1995), Kaitala et al. (1995), de Zeeuw (1998), Chandler et al. (1999), Eyckmans and Tulkens (1999), Germain et al. (2003), Jørgensen and Zaccour (2001), Petrosjan and Zaccour (2003) and Yang (2003). These papers have focused on different issues such as (i) the determination of optimal emissions levels and on the allocation of the resulting total cost to players, (ii) the assessment of the impact of the information structure on emissions and pollution stock, and (iii) the design of mechanisms to insure the sustainability of international environmental agreements. Although the Kyoto Protocol has inspired many of the above-mentioned studies, it is worth noting that up to now the analysis of Kyoto's flexible mechanisms themselves has not yet attracted deserved attention of dynamic games. One exception is Bernard et al. (2002) where an emissions trading differential game is proposed to study the *hot-air* effect.

In a static context, Janssen (1999) has studied the stability of a JI agreement while Lee et al. (1997) provide a game theoretic interpretation for JI in terms of strategic behavior. JI has been also analyzed in a non-technical way in, e.g., Michaelowa (1998), Janssen (2000), Woerdman (2000), Hargrave et al. (1998) for the baseline issue, Dudek and Wiener (1996) for the transaction costs issue, Heintz and Tol (1995) for the uniform mixing property and finally, Begg et al. (2001), Michaelowa (1995, 1996), and Parson and Fisher-Vanden (1999) for more general issues.

[1] The other two mechanisms are emissions trading and clean development.

The differential game considered here involves two non-identical players. Each player attempts to optimize a stream of future earnings over a finite horizon. Three scenarios are studied: the business-as-usual (BAU), the autarky and the JI scenarios. The BAU scenario is a benchmark in which countries do not have to satisfy any environmental constraint. In the autarky game, each country has to satisfy a terminal pollution constraint and invests only locally in environmental projects. In the JIscenario, countries are allowed to invest abroad and receive ERU.

The remainder of the paper is organized as follows; In Section 2, the model and the scenarios are introduced. In Section 3, the different equilibria are derived and discussed. In Section 4 some comparative results are provided. Finally, in Section 5 some concluding remarks are made.

2 Model and scenarios

Three scenarios are considered and will be defined rigorously later on:

- *Business-as-usual scenario*: each country chooses its emissions and local investment strategies so as to optimize a stream of welfares without facing an environmental constraint.
- *Autarky scenario*: each player's optimization problem is as in the previous scenario with the addition of an environmental constraint in the form of a terminal target. In this setting, only local investments in environmental projects are allowed.
- *Joint implementation scenario*: each country faces the same environmental constraint as in the previous scenario but can however invest abroad in environmental projects and collect emissions reduction units for doing so.

In a modelling effort of joint implementation, number of issues which have surfaced in the debate over Kyoto Protocol must be taken care of or at least discussed. These issues are the determination of a baseline level, the complementarity of emissions reductions at home and abroad, the transaction costs, and the leakage. In our framework, the business-as-usual scenario will provide the baseline levels for the two countries. Indeed, the reduction in pollution stock to be achieved by the terminal date is defined as a percentage of the computed pollution stock at the same date under BAU. The complementarity issue is handled when solving the games under the different scenarios, i.e., only equilibria with positive local investments are retained[2]. Although the leakage issue is not explicitly modeled, we will compare net emissions in each country under joint implementation to their counterparts in the autarky scenario. This will enable us to conclude on the presence or not of leakage effects. Finally, we ignore transaction costs for simplicity.

The difference in welfares between the BAU and autarky scenarios provides a measure of the impact of the environmental constraint, would the country have to face it only locally. A comparison of the welfares under the autarky and JI scenarios allows the assessment of

[2] As we will see later on, positivity of local investments is automatically satisfied in all games.

the eventual benefit of joint implementation. Under both autarky and joint implementation scenarios, it is assumed that countries have agreed to comply with an environmental agreement, e.g., Kyoto protocol, and therefore the negotiation process itself is not an issue.

In order to account for the dynamics of pollution, the interactions between the players and for the commitment period in the agreement, we adopt a finite-horizon differential game formalism. We assume that the game involves two players (countries), which we do believe is not too restrictive an assumption given that, most probably, a typical application of the joint implementation mechanism would be of a bilateral nature.

Let $Y_i(t)$ denote the production of country i at $t \in [0, T]$, where T denotes the horizon of the game. This production gives rise to a by-product externality namely (gross) emissions $e_i(t)$. Country i derives from production a net revenue (i.e., gross revenue minus production cost) which is represented by an increasing concave function $R_i(Y_i(t))$. Assuming that production is linearly linked to emissions, the revenue function can thus be expressed in terms of emissions. We assume the following quadratic functional form

$$R_i(e_i(t)) = e_i(t)\left(b_i - \frac{1}{2}e_i(t)\right).$$

For the above revenue function to be concave increasing in emissions, we impose the restriction $e_i(t) \in (0, b_i)$.

Countries can reduce their net emissions by either reducing gross emissions, i.e., by cutting their production, or by reducing net emissions, which can be done by installing less polluting production technologies or by cleaning efforts, or by combining the two options[3].

Denote by $I_{ij}(t)$ the investment effort by country i (called *investor*) in an environmental project in country j (called *host*). Country i incurs an investment cost and a reward in the form of emission reduction units (ERU_i) which are assumed, for simplicity but not unrealistically, proportional to the investment, that is

$$ERU_i(t) = \gamma_j I_{ij}(t),$$

where γ_j is a positive scaling parameter. The latter could also be interpreted as a technological efficiency parameter.For simplicity, we have assumed that the collected reduction units depend on the location where the project is implemented and not on the investor's technology. Countries can also carry out environmental projects at home to reduce gross emissions. Let $Ne_i(t)$ denotes the net emissions in country i, that is

$$\begin{aligned} Ne_i(t) &= e_i(t) - \gamma_i I_{ii}(t) - ERU_j(t) \\ &= e_i(t) - \gamma_i \left(I_{ii}(t) + I_{ji}(t)\right). \end{aligned}$$

[3]Environmental projects are such as, e.g., implementation of carbon-free technologies, substitution of less efficient energies by more efficient ones, replacement of polluting fuels by cleaner ones, increasing the use of renewable resources and energy, afforestation activities, etc.

Remark 1 *We shall assume in the sequel that the net emissions remain positive at any instant of time. The case where net emissions are nonpositive lacks realism. Moreover, the rigorous treatment requires a technical analysis which will not add much to the understanding of investment strategies.*

Denote by $P_i(t)$ the stock of accumulated net emissions in location i. Since we are dealing with a short horizon, we assume that the rate of natural absorption of pollution by sinks and reservoirs is zero. Thus, evolution of the stock of accumulated net emissions is given by

$$\dot{P}_i(t) = e_i(t) - \gamma_i \left(I_{ii}(t) + I_{ji}(t) \right), \quad P_i(0) = P_{i0}.$$

To keep track of the environmental record of country i at any instant of time t, we define a new variable $Ae_i(t)$, which we call *accounted-for-emissions,* as follows

$$\begin{aligned} Ae_i(t) &= e_i(t) - \gamma_i I_{ii}(t) - ERU_i(t) \\ &= e_i(t) - \gamma_i I_{ii}(t) - \gamma_j I_{ij}(t). \end{aligned}$$

Denote by $S_i(t)$ the stock of accumulated accounted-for-emissions of country i, that is

$$S_i(t) = \int_0^t Ae_i(z)dz = \int_0^t \left(e_i(z) - \gamma_i I_{ii}(z) - \gamma_j I_{ij}(z) \right) dz.$$

We shall refer to $S_i(t)$ as the *agreement stock* of player i. This is a state variable and its evolution is described by the following differential equation

$$\dot{S}_i(t) = Ae_i(t) = e_i(t) - \gamma_i I_{ii}(t) - \gamma_j I_{ij}(t), S_i(0) = 0.$$

We assume that each country has accepted to satisfy an environmental target defined in terms of cumulative pollution.This provides a latitude to players to command freely their systems as long as they will end up satisfying the terminal constraint. Denote by $E_i > 0$ the environmental target of player i to be reached at terminal date T. Thus the constraint for player $i, i = 1, 2$, reads

$$S_i(T) = \int_0^T Ae_i(t)dt = \int_0^T \left(e_i(t) - \gamma_i I_{ii}(t) - ERU_i(t) \right) dt \leq E_i.$$

As mentioned in the introduction, the target is defined with respect to a *Business-as-usual* scenario in which countries would not have any environmental constraint. Denote by $e_i^B(t)$ the emissions by country i at instant of time t in this scenario and by τ_i the percentage of reduction assigned to that player in the agreement. The environmental target is thus given by

$$E_i = (1 - \tau_i) \left[P_i^B(T) - P_i(0) \right], \quad 0 \leq \tau_i < 1. \tag{1}$$

where $P_i^B(T)$ is the stock of pollution at T in the business-as-usual scenario.

To see clearly the difference between actual pollution stock $P_i(t)$ and the agreement stock $S_i(t)$, which is an accounting variable, note the following relationships

$$\dot{P}_i(t) - \dot{S}_i(t) = -\gamma_i I_{ji}(t) + \gamma_j I_{ij}(t) = -ERU_i(t) + ERU_j(t), \quad i,j = 1,2, \quad i \neq j$$
$$\sum_{i=1}^{2} \left(\dot{P}_i(t) - \dot{S}_i(t) \right) = 0.$$

Thus, if foreign investments are not allowed, i.e., there is no possibility for collecting emissions-reduction-units, then $\dot{P}_i(t) - \dot{S}_i(t) = 0$ and, of course, one state variable becomes redundant.

To stress the global aspect of pollution control, we shall assume that the damage cost incurred by a country depends on the total stock of pollution $P(t)$ whose evolution is given by

$$\dot{P}(t) = \sum_{i=1}^{2} \dot{P}_i(t) = \sum_{i=1}^{2} \sum_{i \neq j} \left[e_i(t) - \gamma_i \left(I_{ii}(t) + I_{ji}(t) \right) \right], \quad P(0) = P_0. \tag{2}$$

Following the recent estimations obtained in Labriet and Loulou (2003), we suppose that the damage cost is linear and given by $d_i P(t)$, where d_i is a strictly positive parameter. These assumption seems plausible especially in a short-term context as the one we are dealing with. We shall from now on omit the time argument when no ambiguity may arise.

Local investment cost is assumed convex increasing and supposed for tractability quadratic, i.e.,

$$C_i(I_{ii}) = \frac{1}{2} a_i I_{ii}^2, \quad a_i > 0.$$

Foreign investment cost is also assumed quadratic and takes the following form

$$C_i(I_{ij}) = \frac{1}{2}\, a_j \left((I_{jj} + I_{ij})^2 - I_{jj}^2 \right) = \frac{1}{2}\, a_j I_{ij} \left(2I_{jj} + I_{ij} \right), \quad a_j > 0, \quad i \neq j.$$

This functional form captures the idea that the cost for investor i in host country j depends on what the latter is investing at this instant of time. Put differently, investing country has only access to the ERU available in country j after the latter has collected its local ones. The total investment cost of player i is the sum of $C_i(I_{ii})$ and $C_i(I_{ij})$.

We assume a linear salvage cost for the pollution stock given by $\rho_i P(t)$, where ρ_i is a positive parameter. We assume that both players discount their future earnings at zero rate, which is consistent with an intergenerational equity argument.

Country $i, i = 1, 2$, chooses its emissions and investments strategies in order to maximize its welfare functional given by (the superscript J stands for joint implementation):

$$\begin{aligned}
\max_{e_i, I_{ii}, I_{ij}} W_i^J = & \int_0^T \left(e_i \left(b_i - \frac{1}{2}\, e_i \right) - d_i P - \frac{1}{2} \left(a_i I_{ii}^2 + a_j I_{ij} \left(2 I_{jj} + I_{ij} \right) \right) \right) dt \\
& - \rho_i P(t) \\
\text{subject to:} & \qquad\qquad (3) \\
& \dot{S}_i = e_i - \gamma_i I_{ii} - \gamma_j I_{ij}, \quad S_i(0) = 0, \quad S_i(t) \leq E_i, \\
& \dot{P} = \sum_{k=1}^{2} \sum_{k \neq j} \left[e_k(t) - \gamma_k \left(I_{kk}(t) + I_{jk}(t) \right) \right], \quad P(0) = P_0, \\
& 0 \leq e_i \leq b_i, \qquad I_{ii}, I_{ij} \geq 0,
\end{aligned}$$

where E_i is given by (1). Since that this target is defined with respect to the business-as-usual scenario, it is clear that we have to solve the latter problem first.

The optimization problems in the business-as-usual and autarky scenarios obtain easily from the above problem. The autarky scenario stipulates that the countries do still have to satisfy the environmental constraint but foreign investment is not an option. Thus it suffices to set $I_{ij} = 0$ for $i \neq j$ and $I_{ii} \equiv I_i$ in (3) to obtain the autarky optimization problem (the superscript A stands for autarky)

$$\begin{aligned}
& \max_{e_i, I_i} W_i^A = \int_0^T \left(e_i \left(b_i - \frac{1}{2} e_i \right) - d_i P - \frac{1}{2} a_i I_i^2 \right) dt - \rho_i P(t) \\
& \text{subject to:} \qquad\qquad (4) \\
& \qquad \dot{S}_i = e_i - \gamma_i I_i, \quad S_i(0) = 0, \quad S_i(t) \leq E_i, \\
& \qquad \dot{P} = \sum_{i=1}^{2} \left(e_i - \gamma_i I_i \right), \quad P(0) = P_0, \\
& \qquad 0 \leq e_i \leq b_i, \quad I_i \geq 0.
\end{aligned}$$

As mentioned before, in the autarky case, one can replace $\dot{S}_i$ by $\dot{P}_i$.

In the business-as-usual (BAU) scenario, the players do not have to satisfy any environmental constraint. This scenario refers to a situation where no long term commitment is made on reducing collectively the pollution stock. The optimization problem of player i thus reads (where B stands for business-as-usual)

$$\max_{e_i} W_i^B = \int_0^T \left(e_i \left(b_i - \frac{1}{2} e_i \right) - d_i P - \frac{1}{2} a_i I_i^2 \right) dt - \rho_i P(t)$$

subject to: (5)

$$\dot{P} = \sum_{i=1}^{2} \left(e_i - \gamma_i I_i \right), \quad P(0) = P_0,$$

$$0 \leq e_i \leq b_i, \qquad I_i \geq 0.$$

To recapitulate, each of the three above-defined scenarios corresponds to a two-player finite-horizon differential game with emissions and investments as control variables and the pollution and agreement (in autarky and JI) stocks as state variables. Before solving these games, we make the following remarks.

Remark 2 *We assume that these games are played non cooperatively and adopt Nash equilibrium as a solution concept. The rationale behind this assumption is that full coordination of production and investment strategies, i.e., playing a cooperative game, is not possible.*

Remark 3 *It is readily seen that the three differential games are of the linear-state variety. It is well known in this case, (see, e.g., Dockner et al. (2000)), that open-loop Nash equilibria are Markov perfect, the value functions are linear and that the strategies are constant (do not depend on the state).*

Remark 4 *The difference* $W_i^B - W_i^a$ *measures the cost of signing the environmental agreement when there is no possibility of joint implementation. The difference* $W_i^B - W_i^J$ *measures the cost of signing the environmental agreement when the countries are allowed to engage in joint implementation. Thus the "value" of joint implementation, once a country has decided to join the agreement, is given by* $\left(W_i^B - W_i^a\right) - \left(W_i^B - W_i^J\right) = -W_i^a + W_i^J$*. The interest of the approach adopted here is in providing these values and in seeing if joint implementation is Pareto-improving with respect to autarky.*

Remark 5 *Although the environmental target is formulated as an inequality constraint, we shall characterize the equilibria assuming that* $S_i(t) = E_i$ *leaving aside the unlikely case where the players would overcomply with the target.*

3 Equilibria

3.1 Business-as-usual Equilibrium

We start by determining the Nash equilibrium strategies for the business-as-usual scenario. The following proposition summarizes our result.

Proposition 1 *Assuming an interior solution, the unique BAU Nash equilibrium is given by*

$$\begin{aligned} e_i^B &= b_i + d_i\,(t-T) - \rho_i, \qquad i=1,2, &(6)\\ I_i^B &= \frac{\gamma_i}{a_i}(d_i(T-t)+\rho_i), \qquad i=1,2, &(7)\end{aligned}$$

and the pollution stock by

$$P^B = P_0 + t\sum_{k=1}^{2}\left(b_k + \left[d_k\left(\frac{t}{2}-T\right)-\rho_k\right]\left[\frac{a_k+\gamma_k^2}{a_k}\right]\right). \tag{8}$$

Proof. Let $H_i^B\,(e_i, I_i, P, \lambda_i)$ be the Hamiltonian of player i for the *business-as-usual* Nash game defined in (5)

$$H_i^B\,(e_i, I_i, P, \lambda_i) = e_i\left(b_i - \frac{1}{2}e_i\right) - d_iP - \frac{1}{2}a_iI_i^2 + \lambda_i\left(\sum_{i=1}^{2}(e_i-\gamma_iI_i)\right),$$

where λ_i is the adjoint variable to the state pollution equation.

Necessary equilibrium conditions include

$$\begin{aligned} \dot{P}^B(t) &= \sum_{i=1}^{2}(e_i-\gamma_iI_i)\,, \quad P\,(0)=P_0, &(9)\\ \dot{\lambda}_i^B &= d_i, \qquad \lambda_i^B\,(T) = -\rho_i, &(10)\\ \frac{\partial H_i^B}{\partial e_i} &= 0 \Rightarrow e_i^B = b_i + \lambda_i^B, &(11)\\ \frac{\partial H_i^B}{\partial I_i} &= 0 \Rightarrow I_i^B = -\frac{\gamma_i}{a_i}\lambda_i^B. &(12)\end{aligned}$$

The adjoint equation (10) has as its solution

$$\lambda_i^B = d_i\,(t-T) - \rho_i. \tag{13}$$

Equations (11)–(13) lead to equilibrium emissions and investments in (6)–(7). Substituting for control equilibrium values in (9) and integrating gives the equilibrium pollution stock in (8). ■

The results in the above proposition are fairly intuitive. Indeed, emissions are increasing in the revenue parameter b_i and decreasing in the marginal damage cost d_i. Equilibrium investment level satisfies the familiar rule of "marginal cost equals marginal revenue". The latter is defined in terms of the shadow price of pollution stock measuring thus the revenue as the reduction in the damage and salvage costs.

Given that net emissions are given in the BAU case by

$$Ne_i^B(t) = e_i^B(t) - \gamma_i I_i^B(t),$$

it is easy to check that the target E_i can be expressed as function of these net emissions at instant of time $t = T/2$. Indeed,

$$\begin{aligned} E_i &= (1-\tau_i)\left[P_i^B(T) - P_i(0)\right] = T(1-\tau_i)\left[b_i - \left(\frac{Td_i}{2} + \rho_i\right)\left(\frac{a_i + \gamma_i^2}{a_i}\right)\right] \\ &= T(1-\tau_i)\, Ne_i^B(T/2). \end{aligned} \quad (14)$$

Remark 6 *For the solution to be interior we need to have*

$$0 < e_i^B = b_i + d_i(t-T) - \rho_i < b_i, \quad \textit{and} \quad I_i^B > 0, \quad i = 1, 2.$$

For the above conditions to be satisfied, it is easy to check that a sufficient condition is given by

$$b_i > d_i T + \rho_i. \quad (15)$$

This condition has a nice economic interpretation. Indeed, it says that country i's emissions are positive if marginal revenue of emitting evaluated at $e_i = 0$ *is higher than the marginal damage cost time the duration of the game plus the marginal salvage cost.*

Further, we made the assumption that net emissions are positive for all t. It is straightforward to see that this requires

$$b_i > \left(\frac{a_i + \gamma_i^2}{a_i}\right) d_i T + \rho_i. \quad (16)$$

The above condition is stronger than the one in (15). Therefore if net emissions are always positive, then necessarily the solution is interior.

3.2 The Autarky Equilibrium

Recall that in the autarky scenario, countries are assumed to have signed an environmental agreement and thus face a terminal constraint on their cumulative pollution emissions. As mentioned previously, satisfying this constraint can be achieved by cutting production and/or by investing locally in environmental projects. The following proposition characterizes the autarky Nash equilibrium strategies and state trajectories.

Proposition 2 *Assuming an interior solution, the unique Nash equilibrium is given by*

$$e_i^A = \frac{1}{(a_i+\gamma_i^2)}\left(b_i\gamma_i^2+\frac{a_iE_i}{T}\right)+d_i\left(\frac{2t-T}{2}\right), \tag{17}$$

$$I_i^A = \frac{\gamma_i}{a_i}\left(\frac{d_i\,(T-2t)}{2}+\frac{a_i\,(Tb_i-E_i)}{(a_i+\gamma_i^2)\,T}\right), \tag{18}$$

$$S_i^A = t\left(\frac{E_i}{T}+d_i\left(\frac{t-T}{2}\right)\left(\frac{a_i+\gamma_i^2}{a_i}\right)\right), \tag{19}$$

$$P^A = \sum_{i=1}^{2} S_i^a. \tag{20}$$

Proof. The Hamiltonian of problem (4) for player $i, i=1,2$, is given by

$$\begin{aligned} H_i^A\,(e_i,I_i,P,S_i,\lambda_i,\eta_i) &= e_i\left(b_i-\frac{1}{2}e_i\right)-d_iP-\frac{1}{2}a_iI_i^2+\lambda_i\left(\sum_{i=1}^{2}(e_i-\gamma_iI_i)\right) \\ &\quad +\eta_i\,(e_i-\gamma_iI_i)\,, \end{aligned}$$

where λ_i and η_i are the adjoint variables associated to pollution and agreement stocks respectively.

Assuming an interior solution, necessary conditions for a Nash equilibrium are

$$e_i^A = b_i+\lambda_i^A+\eta_i^A, \tag{21}$$

$$I_i^A = \frac{-\gamma_i\left(\lambda_i^A+\eta_i^A\right)}{a_i}, \tag{22}$$

$$\dot{S}_i = e_i^A-\gamma_iI_i^A, \quad S_i\,(0)=0, \quad S_i(t)=E_i, \tag{23}$$

$$\dot{\eta}_i^A = 0, \tag{24}$$

$$\dot{P}^A = \sum_{i=1}^{2}\left(e_i^A-\gamma_iI_i^A\right), \quad P\,(0)=P_0, \tag{25}$$

$$\dot{\lambda}_i^A = d_i, \quad \lambda_i(t)=-\rho_i. \tag{26}$$

Integrating (26) yields

$$\lambda_i^A = d_i\,(t-T)-\rho_i. \tag{27}$$

Substituting for λ_i^A from (27) in (21) and (22) gives emissions and investment as functions of η_i. Substituting them in (23) and integrating gives

$$S_i^A = t\left(b_i+\left(d_i\left(\frac{t}{2}-T\right)-\rho_i+\eta_i^A\right)\left(\frac{a_i+\gamma_i^2}{a_i}\right)\right).$$

Using $S_i^A(T) = E_i$, where the latter is given by (14), yields

$$\eta_i^A = \frac{a_i}{\left(a_i + \gamma_i^2\right)} \left(\frac{E_i}{T} - b_i \right) + \frac{T d_i}{2} + \rho_i. \tag{28}$$

The equilibrium control and state values (17)–(20) are then obtained easily. ■

To interpret the results of this proposition, first note that the shadow price of the pollution stock is the same as in the BAU scenario. This is a by-product of the structure of the game (linear-state). Substituting for E_i by its value from (14) in (28) leads to

$$\begin{aligned}
\eta_i^A &= -\frac{a_i \tau_i}{\left(a_i + \gamma_i^2\right)} \left[b_i - \frac{\left(a_i + \gamma_i^2\right)}{a_i} \left(\frac{d_i T}{2} + \rho_i \right) \right] \\
&= -\frac{a_i \tau_i}{\left(a_i + \gamma_i^2\right)} \left[e_i^B(T/2) - \gamma_i I_i^B(T/2) \right] \\
&= -\frac{a_i \tau_i N e_i^B(T/2)}{\left(a_i + \gamma_i^2\right)} = -\frac{a_i \tau_i E_i}{\left(a_i + \gamma_i^2\right) T \left(1 - \tau_i\right)} < 0.
\end{aligned}$$

Equilibrium condition (24) shows that the adjoint variable to the agreement stock is constant over time. The above shows that its negative value depends actually on all model parameters and that it can be expressed in terms of BAU net emissions at $t = T/2$ or alternatively in terms of the target E_i. Note that η_i^A is equal to zero when there is no constraint on terminal pollution stock (i.e., $\tau_i = 0$) and that $\frac{\partial \eta_i^A}{\partial \tau_i} < 0$. This stipulates that the higher the level of reduction, the higher the shadow price or the cost of the constraint (recall that η_i^A is negative). Yet another way to interpret η_i^A is to express it in terms of the difference in equilibrium gross emissions or investments levels between the BAU and autarky scenarios

$$\begin{aligned}
e_i^A - e_i^B &= \eta_i^A < 0, \\
\frac{a_i}{\gamma_i} \left(I_i^B - I_i^A \right) &= \eta_i^A < 0.
\end{aligned}$$

Thus, an implication of the above proposition is that each country finds it optimal to both decrease its gross emissions and increase its investment in environmental projects in autarky with respect to the BAU equilibrium. The strategy to cope with the environmental target constraint is to act on production and on clean technology and not on only one of these items.

Remark 7 *Since η_i^A and λ_i^A are both strictly negative, a sufficient condition to have an interior solution is provided by*

$$\begin{aligned}
e_i^A &= b_i + \lambda_i^A + \eta_i^A > 0 \\
&\Leftrightarrow \; b_i > \frac{\left(a_i + \gamma_i^2\right)}{\left(a_i \left(1 - \tau_i\right) + \gamma_i^2\right)} \left[\left(1 - \tau_i\right) \rho_i + T d_i \left(1 - \frac{\tau_i}{2} \right) \right].
\end{aligned}$$

For $\tau_i = 0$, we recover the sufficient condition derived in the BAU scenario.

The assumption of positive net emissions at all t requires the following stronger condition

$$b_i > \frac{(a_i + \gamma_i^2)}{a_i (1 - \tau_i)} \left[(1 - \tau_i) \rho_i + T d_i \left(1 - \frac{\tau_i}{2} \right) \right].$$

Thus as in the BAU scenario, if net emissions are always positive, then the solution is necessarily interior.

3.3 The JI Nash equilibrium

In a joint implementation setting, the interesting item is clearly the characterization of foreign investments; which player(s) should invest and what are the driving elements (parameters), are the main questions to be answered. In order to clarify the exposition, we will thus consider non-negativity of investment variables as explicit constraints.

Let $L_i^J (e_i, I_{ii}, I_{ij}, P, S_i, \lambda_i, \eta_i, \mu_{ii}, \mu_{ij})$ be the Lagrangian of the optimization problem (3) of player i, where μ_{ii} and μ_{ij} are Lagrange multipliers appended to nonnegativity of investments constraints:

$$\begin{aligned} L_i^J &= e_i \left(b_i - \frac{1}{2} e_i \right) - d_i P - \frac{1}{2} \left(a_i I_{ii}^2 + a_j I_{ij} \left(2 I_{jj} + I_{ij} \right) \right) + \eta_i \left(e_i - \gamma_i I_{ii} - \gamma_j I_{ij} \right) \\ &+ \lambda_i \left(\sum_{k=1}^{2} \sum_{k \neq j} \left[e_k(t) - \gamma_k \left(I_{kk}(t) + I_{jk}(t) \right) \right] \right) + \sum_{k=1}^{2} \mu_{ik} I_{ik}. \end{aligned}$$

Assuming an interior solution, necessary Nash equilibrium conditions are determined in a coupled way for players i and j because of cross investments and include the following

$$e_i^J = b_i + \lambda_i^J + \eta_i^J, \quad i = 1, 2, \tag{29}$$

$$I_{ii}^J = \frac{-\gamma_i \left(\lambda_i^J + \eta_i^J \right) + \mu_{ii}}{a_i}, \quad i = 1, 2, \tag{30}$$

$$I_{ij}^J = -I_{jj}^J + \left(\frac{-\gamma_j \left(\lambda_i^J + \eta_i^J \right) + \mu_{ij}}{a_j} \right), \quad i, j = 1, 2, \quad i \neq j, \tag{31}$$

$$\mu_{ii} \geq 0, \quad I_{ii} \geq 0, \quad \mu_{ii} I_{ii} = 0, \quad i = 1, 2, \tag{32}$$

$$\mu_{ij} \geq 0, \quad I_{ij} \geq 0, \quad \mu_{ij} I_{ij} = 0, \quad i, j = 1, 2, \quad i \neq j. \tag{33}$$

The adjoint and state variables equations are given by

$$\dot{\eta}_i^J = 0, \tag{34}$$

$$\dot{\lambda}_i^J = d_i, \quad \lambda_i(t) = -\rho_i, \tag{35}$$

$$\dot{S}_i^J = e_i^J - \gamma_i I_{ii}^J - \gamma_j I_{ij}^J, \quad S_i(0) = 0, \quad S_i(T) = E_i, \tag{36}$$

$$\dot{P}^J = \sum_{k=1}^{2} \sum_{k \neq j} \left[e_k^J(t) - \gamma_k \left(I_{kk}^J(t) + I_{jk}^J(t) \right) \right], \quad P(0) = P_0. \tag{37}$$

Integrating (35), leads to

$$\lambda_i^J = d_i (t - T) - \rho_i,$$

and to $\lambda_i^J = \lambda_i^B = \lambda_i^a = \lambda_i$. Again this is due to the linear-state structure of the game.

The following characterizes local investments.

Proposition 3 *In a joint implementation game, if $e_i^J < b_i$, then necessarily local equilibrium investments are strictly positive.*

Proof. Local investments are given by (30)

$$I_{ii}^J = \frac{-\gamma_i \left(\lambda_i + \eta_i^J \right) + \mu_{ii}}{a_i}, \qquad i = 1, 2.$$

We have $e_i^J = b_i + \lambda_i^J + \eta_i^J$. If $e_i^J < b_i$, then $\lambda_i + \eta_i^J \leq 0$. If $I_{ii}^J = 0$, then $\mu_{ii} = \gamma_i \left(\lambda_i + \eta_i^J \right) < 0$, which contradicts the condition in (32) and hence the result. ■

This proposition shows that if a country pollutes, it will not, at any event, rely only on foreign investment to reach its environmental target. Thus our approach captures implicitly the requirement of complementarity of foreign and local investments in a joint implementation endeavour.

Turning now to foreign investments, since that each country can be investor and/or host, we have four situations, with two of them being qualitatively symmetric (those in the second item in the list below), to consider:

- Both players invest abroad. Given that local investments are positive, this would correspond therefore to a case where each country finds it optimal to spread its investments over two locations rather than concentrates them in one place.
- One player invests abroad while the other does not. This is *a priori* the intuitively expected case since that the wisdom on which joint implementation is based is that investments should be channelled toward the most favorable location, in terms of cost, technological efficiency, etc.
- None of the players invest abroad and we are thus back to the autarky equilibrium.

The following proposition rules out the first case of two investing players.

Proposition 4 *In a JI equilibrium, at most one country invests abroad.*

Proof. Assume that $I_{ij}^J > 0$ and $I_{ji}^J > 0$, then $\mu_{ij} = \mu_{ji} = 0$. Using (30), (31) and (33), the equilibrium values for foreign investments become

$$\begin{aligned} I_{ij}^J &= \frac{\gamma_j \left(\lambda_j + \eta_j^J\right)}{a_j} + \left(\frac{-\gamma_j \left(\lambda_i + \eta_i^J\right)}{a_j}\right) \\ &= \frac{\gamma_j}{a_j}\left[\lambda_j - \lambda_i + \eta_j^J - \eta_i^J\right] > 0, \\ I_{ji}^J &= \frac{\gamma_i \left(\lambda_i + \eta_i^J\right)}{a_i} + \left(\frac{-\gamma_i \left(\lambda_j + \eta_j^J\right)}{a_i}\right) \\ &= \frac{\gamma_i}{a_i}\left[\lambda_i - \lambda_j + \eta_i^J - \eta_j^J\right] > 0. \end{aligned}$$

It is easy to see that

$$\begin{aligned} I_{ij}^J &> 0 \Leftrightarrow \lambda_j - \lambda_i > -\eta_j^J + \eta_i^J, \\ I_{ji}^J &> 0 \Leftrightarrow \lambda_j - \lambda_i < -\eta_j^J + \eta_i^J. \end{aligned}$$

The above two inequalities cannot be satisfied simultaneously and thus at equilibrium, at least one of the two foreign investment values must be zero. ■

The following proposition rules out the case where none of the players invest abroad.

Proposition 5 *When foreign investments are possible, autarky is not part of a Nash equilibrium.*

Proof. Assume that $I_{ij}^J = I_{ji}^J = 0$, and $\left[\lambda_j - \lambda_i + \eta_j^J - \eta_i^J\right] \neq 0$,using (30), (31) and (33), the equilibrium foreign investments become

$$\begin{aligned} I_{ij}^J &= \frac{\gamma_j \left(\lambda_j + \eta_j^J\right)}{a_j} + \left(\frac{-\gamma_j \left(\lambda_i + \eta_i^J\right) + \mu_{ij}}{a_j}\right) \\ &= \frac{\gamma_j}{a_j}\left[\lambda_j - \lambda_i + \eta_j^J - \eta_i^J\right] + \frac{\mu_{ij}}{a_j}, \\ I_{ji}^J &= \frac{\gamma_i \left(\lambda_i + \eta_i^J\right)}{a_i} + \left(\frac{-\gamma_i \left(\lambda_j + \eta_j^J\right) + \mu_{ji}}{a_i}\right) \\ &= \frac{\gamma_i}{a_i}\left[\lambda_i - \lambda_j + \eta_i^J - \eta_j^J\right] + \frac{\mu_{ji}}{a_i}. \end{aligned}$$

It is easy to see that $I_{ij}^J = I_{ji}^J = 0$ leads to

$$\begin{aligned}\left[\lambda_j - \lambda_i + \eta_j^J - \eta_i^J\right] &= -\frac{\mu_{ij}}{\gamma_j},\\ \left[\lambda_i - \lambda_j + \eta_i^J - \eta_j^J\right] &= -\frac{\mu_{ji}}{\gamma_i}.\end{aligned}$$

Summing up the two equations yields

$$0 = -\frac{\mu_{ij}}{\gamma_j} - \frac{\mu_{ji}}{\gamma_i} \Leftrightarrow -\frac{\mu_{ij}}{\gamma_j} = \frac{\mu_{ji}}{\gamma_i}.$$

A contradiction to the positivity condition of both multipliers and hence the result. ■

Remark 8 *In the proof of the above proposition, we used implicitly the fact that the game is not **fully** symmetric by assuming that* $\left[\lambda_j - \lambda_i + \eta_j^J - \eta_i^J\right] \neq 0$. *If it were, i.e.,* $b_i = b_j = b, a_i = a_j = a, \gamma_i = \gamma_j = \gamma, E_i = E_j = E, P_{i0} = P_{j0}$, *then other equilibria could materialize. This is however an irrelevant case since complete symmetry lacks of realism and there is, a priori, no point in considering joint implementation when countries are identical.*

To wrap up, we are left with only one possibility, namely that one country invests abroad and the other does not. The following corollary characterizes the foreign investment strategies.

Corollary 1 *Equilibrium foreign investment levels* $\left(I_{ij}^J, I_{ji}^J\right)$ *are either* $\left(\frac{\gamma_j \mu_{ji}}{a_j \gamma_i}, 0\right)$ *or* $\left(0, \frac{\gamma_i \mu_{ij}}{a_i \gamma_j}\right)$.

Proof. The equilibrium conditions are given by

$$\begin{aligned}I_{ij}^J &= \frac{\gamma_j}{a_j}\left[\lambda_j - \lambda_i + \eta_j^J - \eta_i^J\right] + \frac{\mu_{ij}}{a_j},\\ I_{ji}^J &= \frac{\gamma_i}{a_i}\left[\lambda_i - \lambda_j + \eta_i^J - \eta_j^J\right] + \frac{\mu_{ji}}{a_i}.\end{aligned}$$

From the above two propositions, we know that one of these values is positive and the other is zero. Assume first that $I_{ij} > 0$ (and thus $\mu_{ij} = 0$) and $I_{ji} = 0$. The above two equations become

$$I_{ij}^J = \frac{\gamma_j}{a_j}\left[\lambda_j - \lambda_i + \eta_j^J - \eta_i^J\right], \tag{38}$$

$$-\frac{\mu_{ji}}{\gamma_i} = \left[\lambda_i - \lambda_j + \eta_i^J - \eta_j^J\right]. \tag{39}$$

Hence $\left(I_{ij}^J, I_{ji}^J\right) = \left(\frac{\gamma_j \mu_{ji}}{a_j \gamma_i}, 0\right)$. Reversing the roles of the two countries, we obtain the other case. ■

To interpret the above result, let us assume from now on, without any loss of generality, that player i is the investor and player j is the host. From the above corollary, player i's foreign investment is thus given by

$$I_{ij}^J = \frac{\gamma_j \mu_{ji}}{a_j \gamma_i} = \frac{\gamma_j}{a_j}\left[\lambda_j - \lambda_i + \eta_j^J - \eta_i^J\right]. \tag{40}$$

The shadow price of the agreement stock for the investing country obtains by substituting equilibrium emissions and investment values in $\dot{S}_j^J$, integrating and using boundary conditions to get

$$\begin{aligned} \eta_i^J &= \frac{a_j a_i}{\left(a_j a_i + a_j \gamma_i^2 + a_i \gamma_j^2\right)}\left(\frac{E_i}{T} - b_i\right. \\ &\quad \left. + \frac{\gamma_j^2}{a_j}\left(-\frac{d_j T}{2} - \rho_j + \eta_j^A\right)\right) + \frac{d_i T}{2} + \rho_i. \end{aligned} \tag{41}$$

The two above equations with the one providing η_i^A, i.e., (28), lead to the conclusion that the equilibrium level of the foreign investment in environmental project depends on all model's parameters (environmental target, revenue, damage and investment costs, technological efficiency of investments and salvage value parameters) and not only on a simple comparison of investment costs in the two countries. Contrary to the autarky case where we were able to determine easily the sign of the adjoint variable to the agreement stock, the sign of its JI counterpart η_i^J cannot be established readily due to its dependence on the values of all parameters.

Finally, to obtain the pollution state trajectories, it suffices to insert the values of the controls in

$$\begin{aligned} \dot{P}_i^J(t) &= e_i^J(t) - \gamma_i\left(I_{ii}^J(t) + I_{ji}^J(t)\right), \quad P_i(0) = P_{i0}, \\ \dot{P}_j^J(t) &= e_j^J(t) - \gamma_j I_{jj}(t), \quad P_j(0) = P_{j0}, \end{aligned}$$

and to integrate to obtain

$$\begin{aligned} P_j^J &= t\left(b_j + \left(1 + \frac{\gamma_j^2}{a_j}\right)\frac{d_j}{2}(t - T) + \right. \\ &\quad \frac{a_j a_i}{\left(a_j a_i + a_i \gamma_j^2 + a_j \gamma_i^2\right)}\left((1 - \tau_j)\left(b_j - \frac{T d_j}{2} - \rho_j\right) - b_j + \right. \\ &\quad \left.\left. \frac{\gamma_i^2}{a_i + \gamma_i^2}\left((1 - \tau_i)\left(b_i - \rho_i - \frac{T d_i}{2}\right) - b_i\right)\right)\right) + P_{i0}. \end{aligned}$$

$$
\begin{aligned}
P_i^J &= t\left(b_i + \frac{d_i}{2}(t-T) + \frac{a_i}{a_i+\gamma_i^2}\left((1-\tau_i)\left(b_i-\rho_i-\frac{1}{2}Td_i\right)-b_i\right)\right. \\
&\quad + \frac{\gamma_i^2}{a_i}\left(\frac{d_j}{2}(t-T) + \frac{a_j a_i}{\left(a_j a_i + a_i\gamma_j^2 + a_j\gamma_i^2\right)}\left[(1-\tau_j)\left(b_j-\rho_j-\frac{1}{2}Td_j\right)\right.\right. \\
&\quad \left.\left.\left. -b_j + \frac{\gamma_i^2}{a_i+\gamma_i^2}\left((1-\tau_i)\left(b_i-\rho_i-\frac{1}{2}Td_i\right)-b_i\right)\right]\right)\right) + P_{j0}.
\end{aligned}
$$

4 Comparison of Equilibria

This section is devoted to a comparison of the three equilibria in terms of emissions, investments, pollution stocks and welfares. The three questions we wish to answer are:

1. Does JI create a leakage effect?
2. Is JI necessarily Pareto-improving with respect to autarky?
3. Does the imposition of an environmental constraint necessarily deteriorate the individual welfares?

In the appreciation of the comparative results, recall that the pollution stocks in the two constrained scenarios, i.e., P^a and P^J, are by design equal at terminal date T.

Proposition 6 *In the JI game, the host country sets its local investment and emissions at the same levels as those in the autarky equilibrium.*

Proof. Since j is the host country we have $I_{ji} = 0$. Equilibrium emissions and local investment are therefore given by

$$e_j^J = b_j + \lambda_j + \eta_j^J, \tag{42}$$

$$I_{jj}^J = \frac{-\gamma_j\left(\lambda_j + \eta_j^J\right)}{a_j}. \tag{43}$$

The agreement stock evolves according to

$$\dot{S}_j^J = e_j^J - \gamma_j I_{jj}^J, \quad S_j(0) = P_j(0), \quad S_j(T) = E_j.$$

Substituting for emissions and investment and integrating gives

$$S_j^J = t\left(b_j + \left(d_j\left(\frac{t}{2}-T\right) - \rho_j + \eta_j^J\right)\left(\frac{a_j+\gamma_j^2}{a_j}\right)\right).$$

Using the terminal condition $S_i^J(T) = E_i$ allows to obtain

$$\eta_j^J = \frac{a_j}{\left(a_j+\gamma_j^2\right)}\left(\frac{E_j}{T} - b_j\right) + \frac{Td_j}{2} + \rho_j.$$

Clearly $\eta_j^J = \eta_j^A$ (see (28)). It suffices to compare (42)–(43) to (21)–(22) to get the result. ∎

The above proposition implies, among other things, that net emissions in host country are necessarily lower in the JI equilibrium than in the autarky one, thanks to the additional reduction induced by the foreign investment.

Proposition 7 *The pollution stock trajectories compare as follows:*

$$
\begin{array}{llll}
(i) & P_k^B(t) > P_k^A(t), & k=1,2, & \forall t \in [0,T]\,. \\
(ii) & P_j^J(t) - P_j^A(t) < 0, & & \forall t \in [0,T]\,. \\
(iii) & P_i^J(t) - P_i^A(t) > 0, & & \forall t \in [0,T]\,.
\end{array}
$$

Proof.

(i) Straightforward computation gives

$$P_k^B(t) - P_k^A(t) = -t\eta_k^A \left(\frac{a_k + \gamma_k^2}{a_k}\right) > 0$$

since η_k^A is negative.

(ii) Compute

$$P_j^J(t) - P_j^A(t) = t\frac{\gamma_j^2}{a_j}\left[\lambda_i(t) - \lambda_j(t) + \eta_i^J - \eta_j^J\right].$$

It suffices to note from (40) that the above difference can be expressed in terms of the (positive) foreign investment by player i, i.e.,

$$P_j^J(t) - P_j^A(t) = -t\gamma_j I_{ij}^J < 0.$$

(iii) Compute

$$P_i^A(t) - P_i^J(t) = t\left(\frac{a_i + \gamma_i^2}{a_i}\right)\left(\eta_i^A - \eta_i^J\right).$$

Recall that by construction we have

$$
\begin{aligned}
& P_i^J(T) + P_j^J(T) = P_i^A(T) + P_j^A(T) \\
\Leftrightarrow\ & P_j^A(T) - P_j^J(T) = P_i^J(T) - P_i^A(T),
\end{aligned}
$$

and $P_i^A(0) = P_i^J(0)$. From item (ii) we have

$$P_i^A(T) - P_i^J(T) = -T\gamma_j I_{ij}^J(T) = P_j^J(T) - P_j^A(T) < 0.$$

Further,

$$\dot{P}_i^A(t) - \dot{P}_i^J(t) = \left(\frac{a_i + \gamma_i^2}{a_i}\right)\left(\eta_i^A - \eta_i^J\right) = cst.$$

Then the difference $f(t) = P_i^A(t) - P_i^J(t)$ is monotone, with $f(0) = 0$ and $f(T) < 0$. Therefore $f(t) = P_i^A(t) - P_i^J(t) < 0$.

■

By definition of the environmental target, we have that $P_k^B(T) > P_k^A(T), k = 1, 2$. The result in item (i) states that this inequality also holds true for the trajectories. The result in item (ii) is a direct consequence of the fact that the host country keeps its emissions and investment levels in JI at autarky's ones and benefits on the top of that of pollution reduction due to investor's environmental project.

Regarding the investing country, note first that item (iii) of the above proposition implies that $\left(\eta_i^J - \eta_i^A\right) > 0$. Next, it is easy to check that the differences in gross emissions and local investments, between JI and autarky equilibria, are given by

$$\begin{aligned} e_i^J - e_i^A &= \left(\eta_i^J - \eta_i^A\right) > 0, \\ I_{ii}^J - I_i^A &= -\frac{\gamma_i}{a_i}\left(\eta_i^J - \eta_i^A\right) < 0. \end{aligned}$$

The above inequalities implies that

$$\left(e_i^J - \gamma_i I_{ii}^J\right) - \left(e_i^A - \gamma_i I_i^A\right) > 0,$$

meaning that net emissions increase in the investing country in JI with respect to autarky. This is the leakage effect mentioned in the debate over Kyoto flexible mechanisms.Thus, as an answer to our first question, we conclude that JI indeed leads to this leakage effect.

Turning now to the comparison of welfares, the next two propositions provide the comparative results between autarky and joint implementation equilibrium values.

Proposition 8 *For the investing country, the equilibrium JI welfare is at least as its autarky counterpart.*

Proof. From Proposition 6, we know that the host country plays the same strategy under JI and autarky. Given the choice, country i chooses to invest abroad in the JI game rather than to stick to its autarky strategy. Thus the best response of player i to player j's strategy is the one played in the JI game. Therefore $W_i^J \geq W_i^A$. ■

Proposition 9 *If $d_i - d_j > 0$, then $W_j^J > W_j^A$.*

Proof. The welfares are given by

$$\begin{aligned} W_j^A &= \int_0^T \left(e_j^A\left(b_j - \frac{1}{2}e_j^A\right) - d_j P^A - \frac{1}{2}a_j\left(I_j^A\right)^2\right)dt - \rho_j P^A(t), \\ W_j^J &= \int_0^T \left(e_j^J\left(b_j - \frac{1}{2}e_j^J\right) - d_j P^J - \frac{1}{2}\left(a_j\left(I_{jj}^J\right)^2 + a_i I_{ji}^J\left(2I_{ii}^J + I_{ji}^J\right)\right)\right)dt \\ &\quad - \rho_j P^J(t). \end{aligned}$$

Since $e_j^A = e_j^J, I_j^A = I_{jj}^J, I_{ji}^J = 0$, and by construction $P^J(t) = P^A(t)$, the difference in the welfares between autarky and JI of host country becomes

$$\begin{aligned} W_j^A - W_j^J &= d_j \int_0^T \left(P^J - P^A\right) dt \\ &= d_j \left(d_i - d_j\right) \frac{\gamma_i^2}{2a_i} \int_0^T t(t-T)dt \\ &= d_j \left(d_j - d_i\right) \frac{\gamma_i^2 T^3}{12a_i} < 0. \end{aligned}$$

■

The above two propositions show that it suffices to have a higher damage cost in the investing country to be able to conclude on our second question, i.e., that joint implementation is Pareto improving with respect to autarky.

Individual BAU welfares can be higher or lower to their counterparts obtained in the other two scenarios. To show this we provide some numerical experiments. Table 1 provides the parameters values for a series of six different cases. In all the latter, $d_1 \geq d_2$ and country 1 is the investing abroad player in the joint implementation game. The other values of the parameters are chosen arbitrary to the extent that they satisfy all the relationships derived previously and allow to check whether or not imposing an environmental constraint (in autarky and JI games) is necessarily Pareto-deteriorating with respect to business-as-usual.

Table 1: Values of parameters

	a_1	a_2	b_1	b_2	d_1	d_2	ρ_1	ρ_2	γ_1	γ_2	τ_1	τ_2
1	300	300	500	200	10	10	15	30	2	2	0,1	0,02
2	500	500	500	500	50	5	100	5	1	1	0,01	0,01
3	250	200	600	600	5	5	200	10	6	25	0,05	0,01
4	500	500	500	500	5	5	7,5	10	1	1	0,2	0,16
5	500	500	440	500	26	17	170	90	1	1	0,06	0,013
6	250	200	600	600	25	5	200	10	0,5	20	0,05	0,01

Table 2 summarizes the main welfare results, where W^x is total welfare in scenario $x \in \{A, B, J\}$. Since numbers do not have much significance in our theoretical framework and for the purpose of improving readability, only the signs of the results are printed.

The main conclusions that can be derived from these results are the following:

- In cases 1 and 2, at least one player obtains a higher welfare under autarky than under business-as usual scenario. The same is observed when comparing JI to BAU. This shows that imposing an environmental constraint does not necessarily deteriorate all the individual welfares even when the players cannot invest abroad. In these cases,

Table 2: Differences in welfares

	$W_1^A - W_1^B$	$W_2^A - W_2^B$	$W^A - W^B$	$W_1^J - W_1^B$	$W_2^J - W_2^B$	$W^J - W^B$
1	-	+	-	-	+	-
2	+	-	-	+	-	-
3	-	-	-	+	-	+
4	-	-	-	-	-	-
5	+	+	+	+	+	+
6	-	-	-	+	+	+

total welfare under autarky and JI is lower than under BAU counterpart in all the numerical experiments. This is obviously not a rule. Indeed, it suffices to note that in case 3, the JI total welfare is higher than its BAU counterpart.

- Case 4 shows that introducing an environmental constraint deteriorates individual profits with respect to BAU. This is irrespective if the players are allowed or not to invest abroad. Case 5 is a mirror to the previous one. Indeed, here the introduction of the environmental target is Pareto–improving. Case 6 shows that this is the case only if the players play the JI game.

The clear qualitative message of these few experiments is that all possibilities may occur and the ultimate welfare analysis should rest on empirical data.

5 Conclusion

We provided in this paper a differential game formulation of joint implementation in environmental projects. We showed that allowing for foreign investments leads to an improvement in both players (investor and host) payoffs with respect to autarky, and may even improve individual welfares with respect to the business-as-usual scenario.

The following extensions are of interest.

- As it is usually the case in such analysis, the results depend to a certain extent on the functional forms adopted. In our setting, the linearity of the damage cost, although supported by Labriet and Loulou (2003), is crucial. Considering a nonlinear one would have two implications. First, the shadow prices of the pollution stocks obtained in the different scenarios will not be equal and this implies that all welfares ranking would most probably have to be done numerically. Second, the information structure would matter in the sense that the feedback equilibrium will not be degenerate and thus its determination much harder.
- Following the literature, we assumed that the damage cost depends on total pollution stock without taking into account its geographical dispersion. One may conjecture that the leakage effect which occurs under joint implementation is due to the absence of such geographical concern.

- Finally, the extension to n players is needed to analyze investments in environmental projects in a multilateral context.

References

Begg K.G., T. Jackson and S. Parkinson (2001) “Beyond Joint Implementation, Designing Flexibility into Global Climate Policy”, *Energy Policy*, 29, 17–27.

Bernard A., J. Reilly, M. Vielle and L. Viguier (2002) “Russia’s Role in the Kyoto Protocol”, *MIT Program on the Science and Policy of Global Change*, Report 98.

Breton, M., G. Zaccour and M. Zahaf, “A Game Theoretic Formulation of Joint Implementation of Environmental Projects”, mimeo.

Chandler P., H. Tulkens, J.-P. Van-Ypersele and S. Willems (1999) “The Kyoto protocol: an Economic and Game Theoretic Interpretation”, *Economic Theory for the Environment: Essays in Honor of K.-G. Mäler*, Kristrom B., P. Dasgupta and K.-G. Lofgren (eds).

Dockner J.E. and N.V. Long (1993) “International Pollution Control: Cooperative versus Noncooperative Strategies”, *Journal of Environmental Economics and Management*, 25, 13–29.

Dockner J.E., S. Jorgensen S., N.V. Long and G. Sorger (2000) *Differential Games in Economics and Management Science*, Cambridge University Press.

Dudek D.J. and J.B. Wiener (1996) “Joint Implementation, Transaction Costs and Climate Change”, *OECD Economic Outlook*.

Eyckmans J. and H. Tulkens (1999) “Simulating with RICE Coalitionally Stable Burden Sharing Agreements for the Climate Change Problem”, *CORE discussion paper*, 99–26.

Germain, M., P. Toint, H. Tulkens and A. de Zeeuw (2003) “Transfers to sustain dynamic core-theoretic cooperation in international stock pollutant control”, *Journal of Economic Dynamics and Control*, 28(1), 79–99.

Hargrave T., N. Helme and I. Puhl (1998) “Options for Simplifying Baseline Setting for Joint Implementation and Clean Development Mechanism Projects”, Center for Clean Air Policy, Washington DC, USA.

Haurie A. and G. Zaccour (1995) “Differential Game Models of Global Environmental Management”, *Annals of the International Society of Dynamic Games*, 2, 3–24.

Heintz J.R. and R.S.J. Tol (1995) “Joint Implementation and Uniform Mixing”, *Energy Policy*, 23(10), 911–917.

Janssen J. (1999) “Self-Enforcement of Joint Implementation and Clean Development Mechanism Contracts”, *Fondazione Eni Enrico Mattei*, 14–99.

Janssen J. (2000) “Does International Emissions Trading Jeopardize Joint Implementation? Distinguishing the Kyoto Mechanisms for Economic Perspectives”, *Designing Climate Policy, Austrian Council of Climate Change*, Abele, H., T.C. Heller and S.P. Schleicher (eds).

Jørgensen S. and G. Zaccour (2001) "Incentive Equilibrium Strategies and Welfare Allocation in Dynamic Game of Pollution Control", *Automatica*, 3(1), 29–36.

Kaitala V. and M. Pohjola (1995) "Sustainable International Agreements on Greenhouse Warming: a Game Theory Study", *Annals of the International Society of Dynamic Games*, 2, 67–87.

Labriet, M., Loulou, R. (2003) "Coupling Climate Damages and GHG Abatement Costs in Linear Programming Framework", *Environmental Modeling & Assessment*, vol. 8(3), 261–274.

Lee R., J.R. Kahn, G. Marland, M. Russell, K. Shallcross and T.J. Wilbanks (1997) "Understanding Concerns about Joint Implementation", *Joint Institute for Energy and Environment*, Report 97–6.

Martin E.W., R.H. Patrick and B. Tolwinsky (1993) "A Dynamic Game of Transboundary Pollutant with Asymmetric Players", *Journal of Environmental Economics and Management*, 24, 1–12.

Michaelowa A. (1995) "Joint Implementation: a Promising Instrument for Climate Protection", *Intereconomics*, 4, 163–171.

Michaelowa A. (1996) "Incentive Aspects of Joint Implementation of GHG Reduction", *Mitigation and Adaptation Strategies for Global Change*, 1, 95–108.

Michaelowa A. (1998) "Joint Implementation: the Baseline Issue", *Global Environmental Change*, 8(1), 81–92.

Parson E.A. and K. Fisher-Vanden (1999) "Joint Implementation of Greenhouse gas Abatement Under the Kyoto Protocol's Clean development mechanism: its Scope and Limits", *Policy Sciences*, 32(3), 207–224.

Petrosjan L. and G. Zaccour (2003) "Time-Consistent Shapley Value Allocation of Pollution Cost Reduction", *Journal of Economic Dynamics and Control*, 27, 381–398.

van Der Ploeg R. and A. de Zeeuw (1992) "International Aspects of Pollution Control", *Environmental Resource Economics*, 2, 117–139.

Woerdman E. (2000) "Implementing the Kyoto Protocol: why JI and CDM Show More Promise than International Emissions Trading", *Energy Policy*, 28(1), 29–38.

Yang, Z. (2003) "Reevaluation and Renegotiation of Climate Change Coalitions–a Sequential Closed-loop Game Approach", *Journal of Economic Dynamics and Control*, 27(9), 1563–1594.

de Zeeuw, A. (1998) "International Dynamic Pollution Control", *Game Theory and the Environment*, Hanley N. and H. Folmer (eds).