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A Mathematical Model of Economic Growth Connecting Demographic Setting with Controlled Migration

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Abstract. In this paper we propose a mathematical model of migration processes which connects internal demographic setting in a recipient country with external migration under the hypothesis that the migration inflow to this country is affected by the difference between labor productivities of the country and the "outer world" and by the government regulations.

Keywords: migration regulation, mathematical modeling, labor force, economic growth.

INTRODUCTION

International migration is a serious problem which influences all spheres of population's life. Migration has an impact on laboring population, its age structure; migration entails changes in labor and lodging markets. Therefore the importance of mathematical modeling of migration processes is substantial.

Basic methods of migration processes modeling, presented in studies on this topic, are the following:

- econometric models of migration [1];
- analysis of population's development based on stochastic models with continuous time [2];
- model of human capital by G. Becker supposing that a migrant estimates a discounted value of his profits from moving from one country to another [3];
- two-sector models by Harris and Todaro [4];
- modification of Hotelling model of growth and dispersal of populations developed by T. Puu in his work [5].

This paper proposes a mathematical model of migration based on some ideas from these theories. The model connects internal demographic setting in a recipient country with external migration force under the hypothesis that the migration inflow to the country is affected by the difference between labor productivities and the government regulations. The government regulation consists of setting of migration quotas and institutional condition creations. Thus quotas established by the recipient country government, increasing or decreasing of the recipient country attractiveness (implemented again by the recipient country government), migration outflow regulation in donor countries may appear as control actions in our model. The present work deals with the most important kind of migration regulation – recipient country quotas.

We are interested in migration effects on the recipient country economic growth and we completely ignore social problems as, for example, social tensions arising in the recipient country, though they, of course, can influence on migration inflow and the recipient country economy. We consider migration inflow attraction as an economic instrument directed towards the maintenance of the receiving country labor force since it's well known that countries interested in migration inflow have ageing problems and they face with the lack of working population. Also in this paper we don't pay attention to migrant's professional skills and so we don't consider migration influence on the recipient country human capital. The last obstacle can be negotiated, for example, using standard economic growth approach consisting in division of the labor force into skilled and non-skilled.

THE MODEL

We consider a country (we are especially interested in Russian Federation) accepting migrants from the "outer world" which is in some sense averaging-out of all donor countries. We suppose that the immigration grows when the labor productivity of the country increases in comparison with the labor productivity of the "outer world". This

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assumption is empirically confirmed by Russian Federation data and is consistent in some sense with economic theories that pick out a difference in average wage rates as a basic reason of moving from one region to another. Also we assume that the government of the recipient country has an ability to control a migration process (for example by assigning an annual quota on the number of immigrants).

Throughout the article we will interpret the term "population" as the labor force since we are interested only in the part of population involved in a production process. It's well known that usually migrants are capable of working so we assume that after entering the country they supplement the labor force. Also we make the assumption that immigrants stay in the recipient country after arrival. This assumption is not too rigorous since we may think of immigrants as of the migration turnover (so migrants may replace each other within the recipient country).

Denote the whole population of the recipient country, composed of the native population N and the number of immigrants M, by E. Hence $E = N + M$. Then, assuming that all these values depend on time t,

$$\dot{E} = \dot{N} + \dot{M}, \qquad (1)$$

where $\dot{E} = \dfrac{dE}{dt}$.

In traditions of the growth theory the native population increases (or decreases) exponentially. Thus $\dot{N} = \lambda N$, where λ describes the rate of increase (decrease) in the native population. Hence

$$\dot{N} = \lambda N_0 e^{\lambda t}, \qquad (2)$$

where N_0 is the native population at initial time $t_0 = 0$. Note that N_0 may include migrants who came to the country before $t_0 = 0$.

The basic assumption of our model, which consists in the fact that the migration inflow is affected by difference between labor productivities and by government regulations, can be expressed in the two following ideas:

1. in the absence of government regulations the migration rate is assumed to be proportional to the difference between labor productivities in the country and the "outer world" (if we deal with the process of immigration the labor productivity in the country will be higher than the labor productivity in the "outer world");
2. the government establishes the annual quota $\bar{M}$ on the number of migrants who have an ability to enter the country during the year, and if the number of immigrants accumulated from beginning of the year according to the rule 1 runs up to the quota then immediately the migration process will be stopped by the government until the turn of the year.

Formally, this enables us to write the following:

$$\dot{M} = \alpha(z - z_{ex}), \text{if } \alpha\int_{[t]}^{t}(z - z_{ex})dx < \bar{M}, \qquad (3)$$

$$\dot{M} = 0 \text{ otherwise}, \qquad (4)$$

where $z = \dfrac{F(K,E)}{E}$ is the labor productivity within the country, $F(K,E) = aK^{\beta}E^{1-\beta}$ is the two factor Cobb-Douglas production function (surely we are able to use an arbitrary production function, but here we just use the best-known example), K is a capital input, E is a labor input and z_{ex} is the labor productivity in the "outer world" (now we assume that it is constant); $a > 0$, $0 < \beta < 1$ are standard constants and the coefficient $\alpha > 0$ characterizes an attraction to the recipient country from the "outer world", $[x]$ is a floor function notation (so $[t]$ corresponds to beginning of the year).

A capital behavior as in the growth theory is described as

$$\dot{K} = -\delta K + pF(K,E), \qquad (5)$$

where $0 < \delta < 1$ is a depreciation rate of capital and $0 < p < 1$ is a saving rate. δ and p are assumed to be constant. Note that the last supposition regarding p means that we deal with a permanent socio-economic policy.

Using equations described above we conclude that our process is described by the following nonlinear non autonomous system of differential equations:

$$\begin{cases} \dot{E} = \lambda N_0 e^{\lambda t} + \dot{M}, \\ \dot{K} = -\delta K + paK^{\beta} E^{1-\beta}, \end{cases} \quad (6)$$

where $\dot{M}$ is defined by (3)-(4). Note that the first equation of the system has a discontinuous second member.

We interpret K as the capital stock. Model parameters were calibrated so as they in general agree with Russian data; 2005 was taken as the initial year. Analysis of the data sources [6], [7], etc. gives us the following approximated values of parameters: $\alpha = 4$, $a = 0.15$, $\lambda = -0.003$, $N_0 = 73.8$, $z_{ex} = 0.016$. We choose $\beta = \frac{1}{3} \approx 0.33$ since Russia is often considered as the developing country. The ratio of p to δ is chosen so that Russia's GDP without immigration is almost constant during the period of 5 years (figure 1 (a)), according to the data we choose $\delta = 0.06$, the corresponding p is $p = 0.146$. Initial values of the variables are $E(0) = 73.8$, $K(0) = 13.83$. Unit of E is 1 million people (the same for N and M), unit of K is 10^{11} U.S. dollars.

If we set the government quota at the rate 100 thousand migrants per year we will reach a stable growth of GDP during the period under the same initial values and the same values of parameters (figure1 (b)). As we can see in figure 1 (c), the quota rate increasing yields accelerating of the GDP growth.

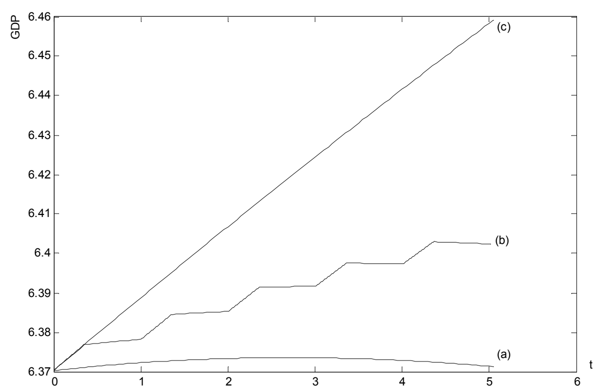

FIGURE 1. GDP Dynamics. (a) $\bar{M} = 0$. (b) $\bar{M} = 0.1$. (c) $\bar{M} = 0.5$.

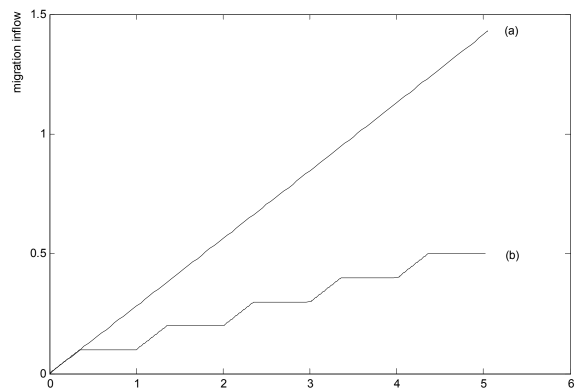

FIGURE 2. Migration Inflow. (a) $\bar{M} = 0.5$. (b) $\bar{M} = 0.1$.

Within the bounds of our model migration inflow can also have a beneficial effect on a capital dynamics.

If there is no government intervention the migration inflow will increase almost linearly (figure 2 (a)), and if the government regulates the migration inflow it will be the staircase function (figure 2 (b)).

Thus the model demonstrates an adequate response to changes in quota rates established by the government. Besides, changes in behavior of system variables in reply to changes in parameter values are expectable and plausible. The main purpose of the work performed was to construct an adequate model and the global aim of our research is to get rid of the assumption (2) and to apply this model to a real demographic setting in the recipient country. We are interested in what quota should establish the recipient country government to avoid the economic stagnation (as mentioned higher, we choose the ratio of p to δ so that recipient country's GDP without immigration is almost constant).

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