doi:10.1111/j.1435-5957.2010.00284.x

Spatial agglomeration with vertical differentiation*

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Received: 30 September 2008 / Accepted: 3 December 2009

Abstract. This paper constructs a two-dimensional framework to take into consideration both horizontal and vertical differentiation. The focus of the paper is on the impact of vertical (quality) differentiation on the location configuration of firms. It shows that while the principle of minimum differentiation in location may be supported, the Pprinciple of maximum differentiation in location can never apply if firms engage in spatially discriminatory pricing. This paper proves that spatial agglomeration gives rise to the unique location equilibrium in both cases where firms charge uniform delivered and mill prices. Moreover, the location equilibria remain unchanged as quality is also endogenously determined.

JEL classification: R3, L15

Key words: Spatial agglomeration, two-dimensional framework, vertical differentiation, competition effect, cost-saving effect

1 Introduction

Hotelling (1929) was the first to propose the theory that two firms manufacturing an homogeneous product would agglomerate at the centre of a linear market based on the assumption of linear transportation costs, a concept which has been termed the *principle of minimum differentiation*. However, D'Aspremont et al. (1979) challenged this principle by indicating that no price equilibrium exists in this case and showed that the two firms would instead locate themselves at the opposite endpoints of the line market under the assumption of quadratic transportation costs. Their counter-assertion has been termed the *principle of maximum differentiation*. Since that time, many regional economists have tried to deduce the conditions under

* We are indebted to anonymous referees for encouraging us to improve our exposition and for offering several suggestions leading to improvements in the substance of the paper. The second author would like to thank the Taiwan National Science Council and Tamkang University for financial support.

which the principle of minimum differentiation can be restored. These researchers include Stahl (1982), who considered harmonious conjectural variations; De Palma et al. (1985) and Rhee et al. (1992), who introduced heterogeneity among both consumers and firms; Anderson and Neven (1991), who assumed that firms engage in Cournot quantity competition instead of Bertrand price competition in the commodity market; Jehiel (1992) and Friedman and Thisse (1993), who adopted price collusion; and Tabuchi (1994), who constructed a model with two dimensions of horizontal differentiation. Tabuchi in particular showed that two firms maximize their distance in one dimension, but minimize their distance in the other dimension. In addition to these researchers, Zhang (1995) imposed a price-matching policy; Mai and Peng (1999) emphasized the importance of the externality-like benefits generated from the exchange of information between firms; Liang and Mai (2006) and Andaluz (2009) focused on the crucial influence arising from the vertical subcontracting of the intermediate product; and Matsushima and Matsumura (2006) analysed a mixed-oligopoly economy.

In addition, Ferreira and Thisse (1996) employed Launhardt's (1885) spatial oligopolistic model to examine the decisions of the firms in regard to optimal quality levels (vertical differentiation), while assuming location (horizontal differentiation) to be exogenously given.[1] They used the transport rate as a measure of quality, with a high (low) transport rate representing a low (high) quality level, and employed a two-stage game in which firms select optimal quality levels in the first stage and then engage in Bertrand price competition in the commodity market during the second stage. Interestingly, they discovered that firms maximize vertical differentiation when the horizontal differentiation is minimized, while minimizing vertical differentiation when horizontal differentiation is maximized. These results are termed the max-min and min-max results hereafter.

Empirically, it can be observed that many industries exist where the choice of location can be regarded as a short-term decision, while quality remains a long-term decision. This kind of setting can be found in Mai and Peng (1999) where location is endogenously determined and R&D (or the quality level) is treated as given.[2] In general, these cases arise in industries where entry costs are lower, namely, firms can easily change their locations with fixed qualities. Examples include various quality levels of restaurants, electronic appliance stores, apparel shops, motels, and so on.

From a theoretical point of view, taking both horizontal differentiation (i.e., location) and vertical differentiation (i.e., qualities) into account allows us to explore the substitutability of quality for location and the strategic interactions between the location-quality combinations that firms provide.[3] To the best of our knowledge, the decision regarding the firms' optimal location, in which the level of quality is exogenously determined, has not yet been touched upon in the literature. This paper aims to fill this gap.

Based on the above analysis, the purpose of this paper is to determine the conditions under which the principle of minimum differentiation can be restored where firms' quality levels (vertical differentiation) are taken into account. Three pricing regimes – discriminatory, uniform delivered and mill pricing – are taken into consideration.

In order to take into account both horizontal and vertical differentiation, we follow Economides (1993) by introducing a two-dimensional model, in which each differentiated product is

[1] According to the definition of Ferreira and Thisse (1996, p. 486), two products are said to be horizontally differentiated when both products have a positive demand whenever they are offered at the same price. Neither product dominates the other in terms of characteristics and heterogeneity in preferences over characteristics explains why both products are present in the market. We can also find a similar definition in Lancaster (1979).

[2] Bonanno and Haworth (1998) also treat quality as a long-term decision, while process R&D remains a short-term decision.

[3] See Economides (1993, p. 236).

defined by one feature of location and one feature of quality. This facilitates the study of the effect of quality differentiation on firms' location decisions.

We show that the equilibrium location is determined by the offsetting of two countervailing effects, namely, the centrifugal competition effect and the centripetal cost-saving effect.[4] The competition effect indicates that as the two firms become more distant from each other, they become more dissimilar and therefore competition is reduced. Accordingly, the two firms tend to distance themselves more from each other to reduce the competition for higher profits by charging higher prices. It is shown in the paper that the introduction of vertical differentiation mitigates the competition effect by enlarging the degree of the differentiation between products. On the other hand, the cost-saving effect reflects firm *i's* desire to move toward the centre in order to save on the cost of transportation. Accordingly, we show that firms agglomerate if the degree of the vertical differentiation between products is high enough, while they separate if the products are less differentiated, as they engage in discriminatory pricing. However, this competition effect is much weakened, leading to the result that spatial agglomeration is the unique location equilibrium, as firms adopt uniform delivered and mill pricing. This result arises because firms charge the same price at every point over the horizontal line segment, leading to the outcome that firms will hardly increase prices and hence profits by moving further apart.

As is shown in this paper, the agglomeration result, under which the degree of vertical differentiation is higher, can be supported by the food service industries, motels, department stores, and apparel shops, etc. We frequently observe that both higher quality restaurants and lower quality food stands in food courts are located close together in many large hotels and department stores, and that motels of various quality levels are located within a small area in numerous venues. Moreover, some department stores, such as Dillards (a high quality store) and JC Penny (a relatively low quality store), and apparel stores, such as Banana Republic (a higher quality store) and The Limited (a relatively low quality store) often agglomerate within the same mall in many towns in the United States. By contrast, the case for dispersal, involving less differentiated products, appears to hold for supermarkets and electronic appliance businesses. There is significant evidence that Walmart and K-mart, as well as Circuit City and local electronic appliance stores, never locate on the same site due to narrow quality differentiation.[5]

The remainder of the paper is organized as follows. Section 2 sets up a spatial model with products exhibiting an exogenously determined degree of vertical differentiation, and analyses the optimal location in the case of discriminatory pricing. Section 3 examines the optimal location in the cases of uniform delivered and mill pricing. Section 4 extends the analysis to the case of location and quality being endogenously determined with a two-stage game. The final section concludes the paper.

2 Spatially discriminatory pricing

Consider a two-dimensional framework, in which the horizontal axis measures the traditional Hotelling (1929) line referred to as the horizontal characteristic, while the vertical axis measures the tastes of consumers based on qualities referred to as the vertical characteristic, as shown in Figure 1.[6] Two firms, denoted as firm 1 and firm 2, are located at x_1 and x_2, with $x_1 \leq x_2$ along a line segment with length $L = 1$ on the horizontal axis. The firms, whose production cost is, for simplicity, assumed to be nil, sell products with vertically differentiated qualities, α_1 and α_2 with

[4] The idea of the competition effect can also be found in Liang et al. (2006).

[5] The pricing policy of Walmart is 'every day low price', and that of Circuit City is 'lowest price guaranteed'. Both pricing policies demonstrate the feature of Bertrand price competition.

[6] Economides (1993) extends the circular model of variety-differentiated products constructed by Salop (1979) to a two-dimensional model, in which both horizontal and vertical differentiation are taken into consideration.

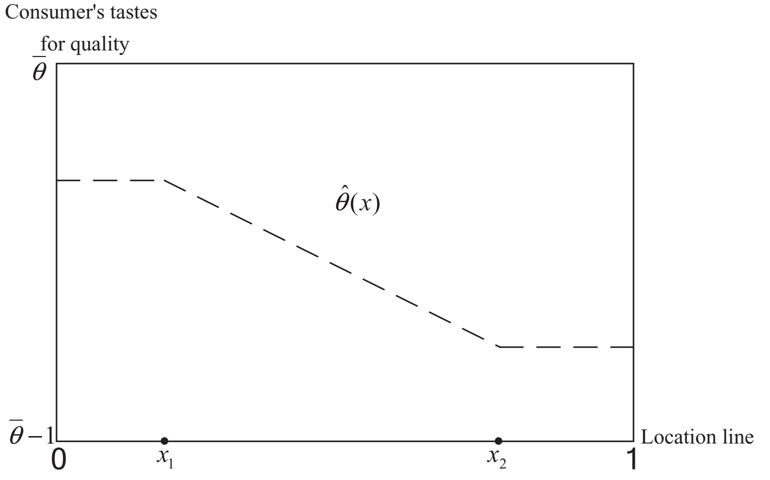

Fig. 1. The two-dimensional framework

$\alpha_1 \leq \alpha_2$, respectively, to consumers.[7] Suppose that quality levels lie within the interval $[\underline{\alpha}, \bar{\alpha}]$, where $\underline{\alpha}$ and $\bar{\alpha}$ denote the lower and upper bounds of the quality levels, respectively. Consequently, the horizontal and vertical characteristics of the products constitute a space of product characteristics over the rectangular $[0,1]\times[\underline{\alpha},\bar{\alpha}]$. In a model with vertically differentiated qualities, there must be heterogeneity in the consumers' willingness to pay for quality, which is captured by assuming that a continuum of consumers is uniformly distributed over the interval $[\underline{\theta}, \bar{\theta}]$ along the vertical axis with unit density at each point of the horizontal line segment.[8] Following Choi and Shin (1992), we assume $\underline{\theta}=\bar{\theta}-1$, where $\bar{\theta}\geq 1$. Thus, these two characteristics lead to a rectangular distribution of consumers over $[0,1]\times[\bar{\theta}-1,\bar{\theta}]$. A firm faces a continuum of consumers with taste $\theta\in[\bar{\theta}-1,\bar{\theta}]$ at each point on the horizontal line segment or a continuum of consumers with different locations for a given taste on the vertical axis. Assume further, that the transport cost function of the product is linear and takes the following form: $T(x - x_i) = t|x_i - x|$, where T is the transport cost, t is the transport rate per unit output per unit distance, x_i denotes firm i's location where consumer x purchases one unit of product, and x represents the location where consumers reside.

Suppose that firms engage in discriminatory pricing, charging different prices to consumers residing in different locations.[9] The indirect utility of a consumer residing at the location with combination (x, θ) and purchasing from firm i can be expressed as:[10]

$$u(x,\theta)=k+\theta\alpha_i-p_i(x),\, i=1,2, \tag{1}$$

[7] The assumption of positive production cost will not change the results derived in this paper. Most of the related literature adopts this assumption. See, for example, Hotelling (1929), D'Aspremont et al. (1979), Anderson and Neven (1991), Tabuchi (1994), Ferreira and Thisse (1996), Mai and Peng (1999), and Matsushima and Matsumura (2006).

[8] Given two products with different qualities, all consumers would prefer the product with higher quality to that with lower quality at the same price. In order for the two firms to survive in the market, the consumers' preferences regarding quality must be heterogeneous.

[9] We assume that the transport rate incurred by firms is lower than that incurred by consumers, implying that there will be no arbitrage among consumers to ensure independent competition between every two points in the market. This assumption fits the reality well. We would like to thank an anonymous referee for pointing out this assumption.

[10] The two-dimensional model will become similar to Hotelling (1929)'s model except that there exists a continuum of consumers whose tastes $\theta\in[\bar{\theta}-1,\bar{\theta}]$ at each site along the Hotelling line segment, if α_2 equals α_1.

where $u(x, \theta)$ is the utility function of the consumer with combination (x, θ); k is the reservation utility of consuming one unit of the commodity; θ reflects the consumers' preferences for quality ranging along the interval $[\bar{\theta}-1, \bar{\theta}]$ with $\bar{\theta}$ being the upper bound of the consumers' tastes; α_i $(i = 1, 2)$ represents the quality level of the product produced by firm i; and $p_i(x)$ is the delivered price charged by firm i at site x.

The taste of the marginal consumer, who is indifferent between buying one unit of the product from either firm, for a continuum of consumers residing at x can be obtained by equating the utility levels from buying from the two firms as follows:[11]

$$\hat{\theta}(x) = [p_2(x) - p_1(x)]/(\alpha_2 - \alpha_1), \tag{2}$$

where $\hat{\theta}(x)$ denotes the taste of the marginal consumer for a continuum of consumers residing at x.

Suppose that the demand for each firm's product is positive at any site x. Thus, each firm's demand function at site x can be derived as:

$$q_1(x) = \{[p_2(x) - p_1(x)]/(\alpha_2 - \alpha_1)\} - (\bar{\theta} - 1), \tag{3.1}$$

$$q_2(x) = \bar{\theta} - \{[p_2(x) - p_1(x)]/(\alpha_2 - \alpha_1)\}. \tag{3.2}$$

Assuming that production costs are zero and the quality cost is fixed, firm i's operating profit function at site x can be expressed as:

$$\pi_i(x) = [p_i(x) - t|x - x_i|]q_i(x), i = 1, 2, \tag{4}$$

where $\pi_i(x)$ denotes firm i's operating profit at site x.

The game in question is a two-stage game, in which firms simultaneously select their optimal locations to maximize profits in the first stage, and then engage in Bertrand competition in the commodity market in the second stage. The sub-game perfect Nash equilibrium can be solved by backward induction, beginning with the final stage. By differentiating (4) with respect to $p_i(x)$, we can derive the profit-maximizing conditions for prices in stage 2. Solving these equations, we have:[12]

$$p_1(x) = [(\alpha_2 - \alpha_1)(2 - \bar{\theta}) + t(2|x - x_1| + |x - x_2|)]/3, \tag{5.1}$$

$$p_2(x) = [(\alpha_2 - \alpha_1)(\bar{\theta} + 1) + t(|x - x_1| + 2|x - x_2|)]/3. \tag{5.2}$$

Substituting (5) into (3), we obtain:

$$q_1(x) = \begin{cases} [(\alpha_2 - \alpha_1)(2 - \bar{\theta}) + t(x_2 - x_1)]/[3(\alpha_2 - \alpha_1)] & \text{if } x \in [0, x_1], \\ [(\alpha_2 - \alpha_1)(2 - \bar{\theta}) + t(x_2 + x_1 - 2x)]/[3(\alpha_2 - \alpha_1)] & \text{if } x \in [x_1, x_2], \\ [(\alpha_2 - \alpha_1)(2 - \bar{\theta}) - t(x_2 - x_1)]/[3(\alpha_2 - \alpha_1)] & \text{if } x \in [x_2, 1]. \end{cases} \tag{6.1}$$

[11] Notice that Equation (2) is derived by assuming that the reservation utility k is sufficiently high such that all consumers buy one unit of a product, namely, the market is covered. However, the main results of the paper remain unchanged if the market is uncovered, namely, some low taste consumers refuse to purchase any product. We use the covered market assumption in order to avoid tedious exposition.

[12] The second-order conditions are satisfied.

$$q_2(x)=\begin{cases}[(\alpha_2-\alpha_1)(\bar{\theta}+1)-t(x_2-x_1)]/[3(\alpha_2-\alpha_1)] & \text{if } x\in[0,x_1],\\ [(\alpha_2-\alpha_1)(\bar{\theta}+1)-t(x_2+x_1-2x)]/[3(\alpha_2-\alpha_1)] & \text{if } x\in[x_1,x_2],\\ [(\alpha_2-\alpha_1)(\bar{\theta}+1)+t(x_2-x_1)]/[3(\alpha_2-\alpha_1)] & \text{if } x\in[x_2,1].\end{cases} \tag{6.2}$$

Recall that $x_1 \leq x_2$ and $\alpha_1 \leq \alpha_2$. Thus, we find from (6.1) that $(2-\bar{\theta})>[t(x_2-x_1)/(\alpha_2-\alpha_1)]\geq 0$, as the output of firm 1 is positive.

Substituting (5) into (2), we can derive the taste of the marginal consumer residing at site x as follows:

$$\hat{\theta}(x)=[(\alpha_2-\alpha_1)(2\bar{\theta}-1)+t(|x-x_2|-|x-x_1|)]/[3(\alpha_2-\alpha_1)],\ x\in[0,1]. \tag{7}$$

Differentiating (7) with respect to x, yields:[13]

$$\partial\hat{\theta}(x)/\partial x=\begin{cases}0 & \text{if } x\in[0,x_1],\\ -2t/3(\alpha_2-\alpha_1)<0 & \text{if } x\in[x_1,x_2],\\ 0 & \text{if } x\in[x_2,1].\end{cases} \tag{8}$$

We see from (8) that given the firms' locations x_1 and x_2, the taste of the marginal consumer remains unchanged for $x \in [0, x_1]$ and $x \in [x_2, 1]$, while taste decreases with respect to x within the interval $[x_1, x_2]$. According to Equations (7) and (8), the relationship between the taste of the marginal consumer and location x along the horizontal line segment is depicted as the broken line in Figure 1. The area above the broken line represents the total output of the high quality firm, while the area below that line denotes the total output of the low quality firm.

Next, we turn to the first stage. Define firm i's aggregate profit function as its aggregate operating profit minus fixed quality costs. Assume that the fixed quality cost function is quadratic and takes the following form: $F(\alpha_i)=\alpha_i^2/2$.[14] This quadratic form of the fixed quality cost function can be frequently found in the literature. See, for example, Dasgupta (1986); D'Aspremont and Jacquemin (1988); Economides (1989, 1993, 1999); Motta (1993); Aoki and Prusa (1996); Aoki (2003); Toshimitsu (2003); Piga and Poyago-Theotoky (2005) and Carlson (2008). It is well recognized that quality improvement involves R&D activity, in which the quadratic form of the fixed quality cost function reflects the following: first, this activity exhibits diseconomies of scale with respect to the amount of the R&D undertaken by the firm, which is reflected by $F'(\alpha_i) > 0$ and $F''(\alpha_i) > 0$; and, second, this activity is independent of output. By substituting (5) and (6) into (4) and then subtracting fixed quality costs from (4), we can derive firm i's reduced aggregate profit function as follows:

$$\begin{aligned}\Pi_1 = \{1/[9(\alpha_2-\alpha_1)]\}\Bigg\{&\int_0^{x_1}[(\alpha_2-\alpha_1)(2-\bar{\theta})+t(x_2-x_1)]^2\,dx\\ &+\int_{x_1}^{x_2}[(\alpha_2-\alpha_1)(2-\bar{\theta})+t(x_2+x_1-2x)]^2\,dx\\ &+\int_{x_2}^{1}[(\alpha_2-\alpha_1)(2-\bar{\theta})-t(x_2-x_1)]^2\,dx\Bigg\}-(\alpha_1^2/2),\end{aligned} \tag{9.1}$$

[13] In the second stage, the firms' locations x_1 and x_2 have been determined in the first stage. We thus have, $\partial x_1/\partial x = 0$ and $\partial x_2/\partial x = 0$.

[14] We are grateful to an anonymous referee for providing us with reasons and related literature to justify the assumption of fixed quality cost. This assumption may be thought of as a situation where firms engage in R&D and advertising activities to improve quality. A justification of this assumption can be found in Dasgupta (1986, p. 523).

$$\begin{aligned}\Pi_2 = \{1/[9(\alpha_2-\alpha_1)]\}\Bigg\{&\int_0^{x_1}[(\alpha_2-\alpha_1)(\bar{\theta}+1)+t(x_1-x_2)]^2\,dx\\ &+\int_{x_1}^{x_2}[(\alpha_2-\alpha_1)(\bar{\theta}+1)+t(-x_1-x_2+2x)]^2\,dx\\ &+\int_{x_2}^{1}[(\alpha_2-\alpha_1)(\bar{\theta}+1)-t(x_1-x_2)]^2\,dx\Bigg\}-(\alpha_2^2/2).\end{aligned} \tag{9.2}$$

Differentiating (9) with respect to x_i, respectively, we obtain:

$$\partial\Pi_1/\partial x_1 = (2t/9)\{-[t(x_2-x_1)(1-x_2+x_1)/(\alpha_2-\alpha_1)]+2[(1/2)-x_1](2-\bar{\theta})\}, \tag{10.1}$$

$$\partial\Pi_2/\partial x_2 = (2t/9)\{[t(x_2-x_1)(1-x_2+x_1)/(\alpha_2-\alpha_1)]+2[(1/2)-x_2](\bar{\theta}+1)\}. \tag{10.2}$$

Note that we can derive the following relationship $1 \ge x_2^* \ge 1/2 \ge x_1^* \ge 0$.[15] Thus, we find from $1 \ge x_2^* \ge 1/2 \ge x_1^* \ge 0$, $\alpha_1 \le \alpha_2$ and $(2-\bar{\theta})>0$ that the first term in the brace on the right-hand side of (10.1) is non-positive, while the second term is non-negative. The profit-maximizing condition for location x_1 equals zero, as these two opposite terms are balanced. Similarly, the first term in the brace on the right-hand side of (10.2) is non-negative, while the second term is non-positive. The profit-maximizing condition for location x_2 equals zero, as these two opposite terms are balanced. As a result, we can derive an interior solution to the equilibrium locations, as the profit-maximizing conditions equal zero. The intuition behind (10) can be stated as follows. The first term in the brace of (10.1) and (10.2) can be referred to as the competition effect, which shows that as the two firms move apart, the horizontal differentiation between the two products is increased, implying that the price competition between firms is mitigated. Consequently, the competition effect attracts firm 1 (2) to move leftward (rightward). Moreover, the competition effect is weakened as the two products become more vertically differentiated (i.e., $\alpha_2 - \alpha_1$, is larger) or as the transport rate is lower. On the other hand, the second term in the braces of (10.1) and (10.2) is denoted as the transportation cost saving effect (for simplicity, the cost-saving effect, hereafter). The cost-saving effect reflects firm *i's* desire to move toward the centre in order to save on the transportation cost. Consequently, equilibrium locations are determined by the offsetting of the competition and cost-saving effects.

The location equilibria are subject to the second-order and stability conditions as follows:

$$\partial^2\Pi_1/\partial x_1^{\,2} = (2t)\{-2(\alpha_2-\alpha_1)(2-\bar{\theta})-t[2(x_2-x_1)-1)]\}/[9(\alpha_2-\alpha_1)] \le 0, \tag{11.1}$$

$$\partial^2\Pi_2/\partial x_2^{\,2} = (2t)\{-2(\alpha_2-\alpha_1)(\bar{\theta}+1)-t[2(x_2-x_1)-1]\}/[9(\alpha_2-\alpha_1)] \le 0, \tag{11.2}$$

$$\begin{aligned}J &= (\partial^2\Pi_1/\partial x_1^2)(\partial^2\Pi_2/\partial x_2^2)-(\partial^2\Pi_1/\partial x_1\partial x_2)(\partial^2\Pi_2/\partial x_2\partial x_1)\\ &=(8t^2)\{2(\alpha_2-\alpha_1)(2-\bar{\theta})(\bar{\theta}+1)+3t[2(x_2-x_1)-1]\}/[81(\alpha_2-\alpha_1)] \ge 0.\end{aligned} \tag{11.3}$$

[15] Recalling that $0 \le x_1 \le x_2 \le 1$ and $\alpha_1 \le \alpha_2$, we find that the first term in the brace on the right-hand side of (10.1) is non-positive. Substituting $x_1 > 1/2$ into (10.1) and attributing this to the relationship $(2-\bar{\theta})>0)$ from (6.1), we can derive $\langle \partial\Pi_1/\partial x_1|_{x_1>1/2}\rangle < 0$, implying that the optimal location x_1* lies within [0, 1/2]. Similarly, the first term in the brace on the right-hand side of (10.2) is non-negative. Substituting $x_2 < 1/2$ into (10.2) gives $\langle \partial\Pi_2/\partial x_2|_{x_2<1/2}\rangle > 0$, implying that the optimal location x_2* lies within [1/2, 1]. As a result, we prove that $1 \ge x_2^* \ge 1/2 \ge x_1^* \ge 0$.

In addition, the location equilibria should fulfil the market-serving condition, which requires the output of each firm at its remote endpoint shipped from its equilibrium location to be positive. This can be described as follows:[16]

$$q_1(1; x_1, x_2) = \left[(\alpha_2 - \alpha_1)(2-\bar{\theta}) - t(x_2 - x_1)\right] / [3(\alpha_2 - \alpha_1)] > 0, \tag{12.1}$$

$$q_2(0; x_1, x_2) = \left[(\alpha_2 - \alpha_1)(1+\bar{\theta}) - t(x_2 - x_1)\right] / [3(\alpha_2 - \alpha_1)] > 0. \tag{12.2}$$

Solving (10) and the corresponding three conditions, we obtain the following location equilibria (see Appendix for the proof):

$$x_1{}^A = x_2{}^A = 1/2, \text{ if } \alpha_2 - \alpha_1 \geq \Delta_3{}^A, \tag{13.1}$$

$$\begin{aligned} x_1{}^D &= \left[2(\alpha_2 - \alpha_1)(2+3\bar{\theta} - \bar{\theta}^3)/(9t)\right] + \left[(1-2\bar{\theta})/6\right], \text{ and} \\ x_2{}^D &= \left[2(\alpha_2 - \alpha_1)(-4+3\bar{\theta}^2 - \bar{\theta}^3)/(9t)\right] + \left[(7-2\bar{\theta})/6\right], \text{ if } \Delta_3{}^D \leq \alpha_2 - \alpha_1 \leq \Delta_3{}^A. \end{aligned} \tag{13.2}$$

Where the superscript '*A*' ('D') denotes the variables associated with the case of the agglomeration (dispersal) equilibrium, respectively.

Equations (13.1) and (13.2) show that central agglomeration arises only if the degree of vertical differentiation is sufficiently large, say, $(\alpha_2 - \alpha_1) \geq \Delta_3{}^A$, while spatial dispersion arises as the degree of vertical differentiation lies in between $[\Delta_3{}^D, \Delta_2{}^D]$, namely, $\Delta_3{}^D \leq (\alpha_2 - \alpha_1) \leq \Delta_3{}^A$. The intuition behind this result can be stated as follows. We have shown that the location equilibrium is jointly determined by the competition and cost-saving effects. Moreover, the higher the degree of vertical differentiation is, the weaker will be the competition effect. Therefore, as the degree of vertical differentiation is no less than the critical value $\Delta_3{}^A$, the competition effect is dominated by the cost-saving effect so that the two firms agglomerate at the centre of the horizontal line segment. On the contrary, as the degree of vertical differentiation is higher than the critical value $\Delta_3{}^A$, the competition effect outweighs the cost-saving effect so that the two firms start to separate.

Based on the above analysis, we can establish:[17]

Proposition 1. *Assuming that firms engage in discriminatory pricing, it follows that*:

(*i*) *firms agglomerate at the centre of the horizontal line segment as the degree of vertical differentiation is high enough, namely,* $(\alpha_2 - \alpha_1) \geq \Delta_3{}^A$*; and*

[16] Note that the definition of the market-serving condition in the paper is different from that of Anderson and Neven (1991) (hereafter AN). AN's market-serving condition is implemented in the second (output) stage, which requires that, *for any given location*, *even if the firms are at the opposite endpoints*, each firm's sales to its remote endpoint be positive. It should be noted that the location equilibrium, in which firms are at the opposite endpoints, can never occur under the linear transport cost function, implying that AN's condition is unnecessarily restrictive. Next, the rigorousness of AN's condition has been challenged by Chamorro-Rivas (2000) and Yang et al. (2008). Both papers point out that firms have selected their locations in stage 1 before the prices are solved in stage 2. Thus, the market-serving condition should be implemented in the first stage. Accordingly, they argue that AN's market-serving condition should be modified so that, *at the equilibrium location*, each firm's sales to its remote market are positive.

[17] Intuitively, as the transport cost is quadratic in distance, the competition effect (i.e., the centrifugal force) will become stronger than what it will be in the case of the linear transport cost function. Thus, it is harder to restore the principle of minimum differentiation in the former case than in the latter case. However, we calculate and then find that firms agglomerate at the centre of the horizontal line segment as the degree of vertical differentiation is sufficiently large under the quadratic transport cost function, while separate otherwise. The calculations are not reported here to save space, but they are available from the authors upon request. Consequently, the results described in Proposition 1 are robust irrespective of the transport cost function being linear or quadratic in distance whenever firms engage in spatially discriminatory pricing.

(*ii*) *spatial dispersion emerges as the degree of vertical differentiation lies in a medium range, say,* $\Delta_3^D \leq (\alpha_2 - \alpha_1) \leq \Delta_3^A$.

Proposition 1 shows a striking result in that the principle of minimum differentiation can be restored, as the degree of vertical differentiation is high enough. Moreover, it should be noted that the results derived in Proposition 1 are significantly different from those in Economides (1989), in which the principle of maximum differentiation in location is the only location equilibrium and firms choose an identical quality level.[18] It can be frequently observed in the real world that firms can either agglomerate or locate separately at various distances from each other. As mentioned in the Introduction, examples supporting the agglomeration result include various quality levels of restaurants, food stands in food courts, motels, department stores and apparel shops, while firms in the supermarket and electronic appliance store business are usually separated. When contrasted with the fact that the relevant literature can only explain the case where the principle of maximum differentiation in terms of location applies, this paper can explain the real-world phenomena quite well.

Next, we demonstrate the invalidity of the principle of maximum differentiation in the context of the model above. First of all, we examine this principle as the interior location equilibrium is reached, which can be done via (13.2). Recall that $2-\bar{\theta}>0$, $\underline{\theta}=\bar{\theta}-1\geq 0$, and $\alpha_1 \leq \alpha_2$. We find from (13.2) that $x_2^D=\left[2(\alpha_2-\alpha_1)\left(-4+3\bar{\theta}^2-\bar{\theta}^3\right)/(9t)\right]+\left[\left(7-2\bar{\theta}\right)/6\right]<5/6$, which indicates that firm 2 would never locate at the right end of the horizontal line segment.[19] Thus, the principle of maximum differentiation will never emerge in this case. Second, we examine this principle when a corner solution for location equilibrium occurs. This will arise whenever the conditions $\partial\Pi_1/\partial x_1 < 0$ and $\partial\Pi_2/\partial x_2 > 0$ hold. We can calculate from (10.1) that the former condition holds if $(\alpha_2-\alpha_1)>\left\{\left[t(x_2-x_1)(1-x_2+x_1)\right]/\left[(1-2x_1)\left(2-\bar{\theta}\right)\right]\right\}\equiv\Delta_1^c$ and from (10.2) that the latter condition holds if $(\alpha_2-\alpha_1)<\left\{\left[t(x_2-x_1)(1-x_2+x_1)\right]/\left[(2x_2-1)\left(1+\bar{\theta}\right)\right]\right\}\equiv\Delta_2^c$. We find that $\Delta_1^c=\Delta_2^c=0$ as $x_1=0$ and $x_2=1$. Thus, the conditions hold only if $\alpha_2-\alpha_1<0$, which contradicts the assumption $\alpha_1 \leq \alpha_2$ and therefore excludes the possibility of the principle of maximum differentiation.

Based on an analysis of Equation (13), we can depict the relationship between firms' location equilibria and the degree of vertical differentiation in Figure 2. The loci D_1AE represent firm 1's location equilibrium, while the loci D_2AE denote firm 2's location equilibrium.[20] Figure 2 shows that since the degree of vertical differentiation, $\alpha_2 - \alpha_1$, is no less than Δ_3^A, both firms agglomerate at the centre of the horizontal line segment, while they remain apart whenever the degree of vertical differentiation lies within the interval (Δ_3^D, Δ_3^A).

Proposition 2. *Assuming that firms engage in discriminatory pricing, the principle of maximum differentiation can never occur.*

By assuming location is determined prior to quality in a one-dimensional model with mill pricing, Ferreira and Thisse (1996) derive the max-min and min-max results. Moreover, by

[18] In addition, our results are also sharply different from those derived in a circular model such as Economides (1993), Pal (1998), Yu and Lai (2003), Gupta et al. (2004) and Yu (2007), in which the principle of maximum differentiation in terms of location is the unique equilibrium in a two-firm case. We are grateful to an anonymous referee for pointing out this difference.

[19] This arises because $\alpha_2-\alpha_1 \geq 0$, $\left(-4+3\bar{\theta}^2-\bar{\theta}^3\right)<0$, and $\left[\left(7-2\bar{\theta}\right)/6\right]<5/6$.

[20] Manipulating eq. (13.2), we obtain $\partial x_1^D/\partial(\alpha_2-\alpha_1)=2\left(2+3\bar{\theta}-\bar{\theta}^3\right)/(9t)>0$, $\partial x_2^D/\partial(\alpha_2-\alpha_1)=2\left(-4+3\bar{\theta}^2-\bar{\theta}^3\right)/(9t)<0$, and $\partial^2 x_i^D/\partial(\alpha_2-\alpha_1)^2=0$, $i = 1, 2$.
Accordingly, we find that D_1A and D_2A are linear, the slope of D_1A (D_2A) is positive (negative) and D_2A is steeper than D_1A since the absolute value of the slope of D_2A is larger than that of D_1A.

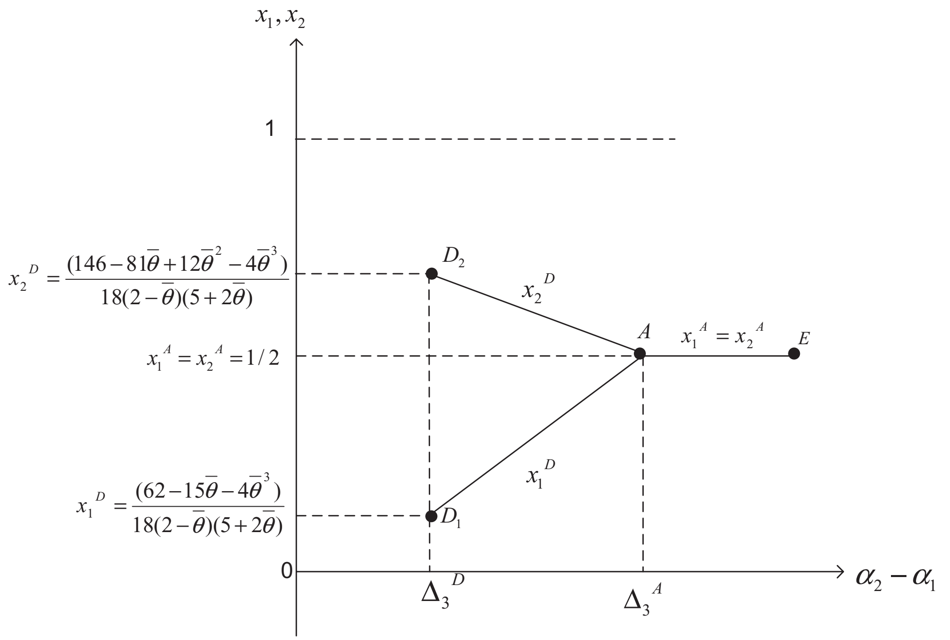

Fig. 2. The relationship between the firms' location equilibrium and the degree of vertical differentiation

letting location be endogenously determined, Economides (1989) finds that firms locate themselves as far apart as possible.

By manipulating (13.2), we can derive the distance (degree of horizontal differentiation) between the two firms' location equilibria for the case of spatial dispersion as follows:

$$x_2^D - x_1^D = 1 - \{2(\alpha_2 - \alpha_1)(2-\bar{\theta})(\bar{\theta}+1)/(3t)\}. \tag{14}$$

Differentiating the distance with respect to $(\alpha_2 - \alpha_1)$ and t, respectively, we obtain:

$$\partial(x_2^D - x_1^D)/\partial(\alpha_2 - \alpha_1) = -(2-\bar{\theta})(\bar{\theta}+1)/(3t) < 0, \tag{15.1}$$

$$\partial(x_2^D - x_1^D)/\partial t = 2(\alpha_2 - \alpha_1)(2-\bar{\theta})(\bar{\theta}+1)/(3t^2) > 0. \tag{15.2}$$

We see from (15) that the distance between the two firms declines as the degree of vertical differentiation rises, while it increases as the transport rate rises. Intuitively, the products become more differentiated leading to a weaker competition effect as the degree of the vertical differentiation becomes higher. The upshot is that the two firms approach each other spatially. On the other hand, the competition effect strengthens due to higher delivered prices as the transport rate rises. This induces firms to move further apart, while they charge higher prices and earn higher profits. Accordingly, we can establish Proposition 3.

Proposition 3. *Assuming that firms engage in discriminatory pricing, firms locate further apart in the dispersal equilibrium as the degree of vertical differentiation falls or the transport rate rises.*

3 Uniform delivered and mill pricing

In this section, we will examine the firms' location equilibria as firms undertake uniform delivered and mill pricing in the commodity market. We first discuss the case of uniform delivered pricing. Firms charge the same delivered price at each point on the horizontal line segment in this case. Thus, the indirect utility of a consumer, who purchases from firm *i*, can be expressed as:

$$u(x,\theta) = k + \theta\alpha_i - p_i^{\,u}, i = 1, 2, \tag{16}$$

where the superscript '*u*' denotes the variables associated with the case of uniform delivered pricing.

The marginal consumer, who is indifferent to buying one unit of product from either firm, following (16), acts so as to satisfy:

$$\hat{\theta}^u = (p_2^{\,u} - p_1^{\,u})/(\alpha_2 - \alpha_1) \tag{17}$$

Firms' demand functions under the case of uniform delivered pricing are therefore equal to:

$$q_1^{\,u} = [(p_2^{\,u} - p_1^{\,u})/(\alpha_2 - \alpha_1)] - (\bar{\theta} - 1), \tag{18.1}$$

$$q_2^{\,u} = \bar{\theta} - [(p_2^{\,u} - p_1^{\,u})/(\alpha_2 - \alpha_1)]. \tag{18.2}$$

The firm's aggregate profit function can be obtained by integrating its operating profit at each point along the horizontal line segment minus fixed quality costs, and can be expressed as:

$$\Pi_i^{\,u} = \int_0^1 (p_i^{\,u} - t|x - x_i|) q_i(x)\,dx = q_i^{\,u}\{p_i^{\,u} - t[x_i^2 - x_i + (1/2)]\} - (\alpha_i^2/2), i = 1,2. \tag{19}$$

In order to save space, we skip the procedure for stage 2 and jump directly to the profit-maximizing conditions for location in stage 1. These conditions are as follows:

$$\partial\Pi_i^{\,u}/\partial x_i = 4t(\alpha_2 - \alpha_1) q_1^{\,u}[(1/2) - x_i] = 0, i = 1, 2. \tag{20}$$

Note that the competition effect in (20) is much weakened. This arises because firms charge the same price for every point along the horizontal line segment, leading to the result that it is hard for firms to increase price and profits by locating themselves further away from each other. Thus, the cost-saving effect definitely outweighs the competition effect so that firms will locate themselves at the centre of the horizontal line segment to minimize transport costs. This result can be supported by solving (20) while considering all three restrictions, namely, the second-order, the stability and the market serving conditions, as follows:

$$x_i^{\,u} = \frac{1}{2}, i = 1, 2. \tag{21}$$

Accordingly, we have the following proposition.

Proposition 4. *Assuming that firms engage in uniform delivered pricing, central agglomeration is the unique location equilibrium if the degree of vertical differentiation is greater than zero.*

This result differs sharply from that derived in the case of discriminatory pricing, in which spatial dispersion occurs whenever the degree of vertical differentiation is sufficiently low.

Next, we turn to the case of mill pricing. The indirect utility of a consumer residing at site x can be rewritten as:

$$u(x,\theta) = k + \theta\alpha_i - p_i^f - t|x - x_i|, \tag{22}$$

where the superscript 'f' denotes the variables associated with the case of mill pricing.

Similarly, we jump directly to the profit-maximizing conditions for location in stage 1 by following the same procedures as those in the case of uniform delivered pricing. These conditions are as follows:

$$\partial\Pi_i^f/\partial x_i = 4t(\alpha_2 - \alpha_1)q_1^u[(1/2) - x_i] = 0, i = 1, 2. \tag{23}$$

Solving (23) with due consideration to all three constraints yields the firms' optimal locations as follows:

$$x_i^f = \frac{1}{2}, i = 1,2. \tag{24}$$

We see from (24) that central agglomeration is the unique location equilibrium. The same intuition stated in the case of uniform delivered pricing applies to this case. As a consequence, we can establish the following proposition.

Proposition 5. *Assuming that firms charge mill pricing, central agglomeration is the unique location equilibrium if the degree of vertical differentiation is greater than zero.*

4 Extension

In this section, we extend our analysis to the case of quality being endogenously determined with a two-stage game, in which quality and location are selected in the first stage, while firms engage in Bertrand price competition in the second stage. Likewise, three pricing schemes are taken into account.

4.1 Spatially discriminatory pricing

In this subsection, we explore the firms' location and quality selections as firms engage in spatially discriminatory pricing in the commodity market. Similarly, the game is solved by backward induction. Since the equilibrium prices derived in stage 2 are identical to those of Equation (5), we shall skip this procedure and jump directly to the first stage to discuss the firms' selections regarding locations and quality. Differentiating (9) with respect to x_i and α_i, respectively, yields the profit-maximizing conditions for locations and quality. We find that the profit-maximizing conditions for locations are identical to those of Equation (10). Therefore, we only report the profit-maximizing conditions for qualities as follows:

$$\partial\Pi_1/\partial\alpha_1 = -\left[(2-\bar{\theta})^2/9\right] + \left\{t^2(x_2 - x_1)^2[3 - 2(x_2 - x_1)]\Big/\left[27(\alpha_2 - \alpha_1)^2\right]\right\} - \alpha_1 = 0, \tag{25.1}$$

$$\partial\Pi_2/\partial\alpha_2 = \left[(1+\bar{\theta})^2/9\right] - \left\{t^2(x_2 - x_1)^2[3 - 2(x_2 - x_1)]\Big/\left[27(\alpha_2 - \alpha_1)^2\right]\right\} - \alpha_2 = 0. \tag{25.2}$$

Solving Equation (10) gives two possible location equilibria, central agglomeration and spatial dispersion, as shown in Equations (13.1) and (13.2).

We first discuss the optimal quality levels when firms agglomerate at the market centre. By substituting (13.1) into (25.1), we find that $\partial\Pi_1/\partial\alpha_1 = \left[(2-\bar{\theta})^2/9\right] - \alpha_1 < 0$, implying that firm 1 will select the lower bound value as its optimal quality level. On the other hand, by substituting (13.1) into (25.2), we can calculate firm 2's optimal quality level. Consequently, the firms' optimal quality levels when firms agglomerate at the centre of the horizontal line segment can be solved as follows:

$$\alpha_1{}^* = \underline{\alpha}, \tag{26.1}$$

$$\alpha_2{}^* = (1+\bar{\theta})^2/9. \tag{26.2}$$

We see from (26.2) that firm 2's optimal quality level can be greater than the upper bound value of the quality level, $\alpha_2^* \geq \bar{\alpha}$, if this value is not sufficiently high. Thus, we can conclude from Equations (10) and (26) that the min-max result may occur, if this upper bound value is not sufficiently high.

Next, by substituting (13.2) into (25), we can derive firm *i*'s optimal quality levels when the dispersal equilibrium emerges. However, as the derivations of these optimal quality levels are tedious to explain and the focus of the paper is on the firms' location choices, it does not matter if we do not show how they are derived. Thus, we do not report them here. Instead, we express the relationship between these optimal quality levels. It is found from (25) that the relationship can be described as follows:

$$\alpha_2 = [(2\bar{\theta}-1)/3] - \alpha_1. \tag{27}$$

We now turn to examine whether the principle of maximum differentiation can occur when qualities are endogenously determined. The distance between the two firms can be derived by (14) as $x_2{}^D - x_1{}^D = 1 - \left[2(\alpha_2 - \alpha_1)(2-\bar{\theta})(\bar{\theta}+1)/(3t)\right]$. Recall that $2-\bar{\theta} > 0$. Moreover, we find from the second-order conditions for qualities that the quality difference between firms must be greater than zero, namely, $\alpha_2 - \alpha_1 > 0$.[21] Accordingly, we can find that the distance between the two firms must be less than one, namely, $x_2{}^D - x_1{}^D < 1$. Thus, the principle of maximum differentiation can never occur, and neither can the Max-Min result occur, either.

Based on the above analysis, we can establish the following proposition.

Proposition 6. *Assuming that firms charge spatially discriminatory prices and that quality and location are endogenously determined, it follows that*:

(*i*) *firm 1 chooses the lower bound value as its optimal quality level while firm 2 selects an optimal level, say,* $(1+\bar{\theta})^2/9$, *when firms agglomerate at the centre of the horizontal line segment;*

(*ii*) *the relationship between the two firm's quality levels is* $\alpha_2 = [(2\bar{\theta}-1)/3] - \alpha_1$ *when spatial dispersion emerges. Moreover, the principle of maximum differentiation can never occur; and*

(*iii*) *the min-max result may occur if the upper bound value of the quality level is not sufficiently high. However, the max-min result will never occur.*

[21] The second-order conditions for qualities require that $\partial^2\Pi_i/\partial\alpha_i^2 < 0$, $i = 1, 2$, resulting in the following inequality $27(\alpha_2 - \alpha_1)^3 > 2t^2(x_2 - x_1)^2[3 - 2(x_2 - x_1)] \geq 0$, $i = 1, 2$.

4.2 Uniform delivered and mill pricing

In this subsection, we analyse the endogenous locations and qualities when firms adopt uniform delivered and mill pricing. We first discuss the case of uniform delivered pricing. It can be easily found that the profit-maximizing conditions for locations are the same as in Equation (20) and can be solved independently to yield $x_i^u = 1/2$, $i = 1, 2$. Thus, firms will choose to agglomerate at the centre of the horizontal line segment. On the other hand, firms' profit-maximizing conditions for qualities can be derived as follows:

$$\partial \Pi_1^u / \partial \alpha_1 = \left[m_1^2 / 9(\alpha_2 - \alpha_1)^2 \right] - \{ 2(2-\bar{\theta}) m_1 / [3(\alpha_2 - \alpha_1)] \} - \alpha_1, \tag{28.1}$$

$$\partial \Pi_2^u / \partial \alpha_2 = -\left[m_2^2 / (\alpha_2 - \alpha_1)^2 \right] + \{ 2(1+\bar{\theta}) m_2 / [3(\alpha_2 - \alpha_1)] \} - \alpha_2, \tag{28.2}$$

where $m_1 = [(\alpha_2 - \alpha_1)(2-\bar{\theta}) - t(x_2 - x_1)(1 - x_2 - x_1)]/3$, and $m_2 = [(\alpha_2 - \alpha_1)(1+\bar{\theta}) + t(x_2 - x_1)(1 - x_2 - x_1)]/3$.

Substituting $x_i^u = 1/2$, $i = 1, 2$ into Equation (28), we can find that the optimal quality levels are the same as those of Equation (26).

Next, we turn to the case of mill pricing. Following the same procedure, we find that the agglomeration equilibrium is the unique location equilibrium, namely, $x_i^f = 1/2$, i = 1, 2. Similarly, the firms' profit-maximizing conditions for qualities are:

$$\partial \Pi_1^f / \partial \alpha_1 = \left[m_1^2 / 9(\alpha_2 - \alpha_1)^2 \right] - \{ [2(2-\bar{\theta}) m_1 / [3(\alpha_2 - \alpha_1)] \} - \alpha_1, \tag{29.1}$$

$$\partial \Pi_2^f / \partial \alpha_2 = -\left[m_2^2 / (\alpha_2 - \alpha_1)^2 \right] + \{ [2(1+\bar{\theta}) m_2 / [3(\alpha_2 - \alpha_1)] \} - \alpha_2, \tag{29.2}$$

where $m_1 = [(\alpha_2 - \alpha_1)(2-\bar{\theta}) - t(x_2 - x_1)(1 - x_2 - x_1)]/3$, and $m_2 = [(\alpha_2 - \alpha_1)(1+\bar{\theta}) + t(x_2 - x_1)(1 - x_2 - x_1)]/3$.

Substituting $x_i^f = 1/2$, $i = 1, 2$ into Equation (29), we also find that the optimal quality levels are the same as those of Equation (26).

Based on the above analysis, we show that the agglomeration equilibrium is the only location equilibrium and that Equation (26) holds under both cases of uniform delivered and mill pricing. Thus, the same results, in which the min-max result may occur if this upper bound value is not sufficiently high while the max-min can never occur, applies to these two cases. Accordingly, we have the following proposition.

Proposition 7. *Assuming that firms engage in uniform delivered and mill pricing and that quality and location are endogenously determined, it follows that*:

(*i*) *central agglomeration is still the unique location equilibrium. Moreover, firm 1 chooses the lower bound value as its optimal quality level while firm 2 selects an optimal level, say,* $(1+\bar{\theta})/9$*; and*

(*ii*) *the Min-Max result may occur if the upper bound value of the quality level is not sufficiently high while the Max-Min can never occur.*

5 Concluding remarks

This paper has constructed a two-dimensional framework that takes into account both features of horizontal and vertical differentiation. The analysis in the paper has shown that firms' location decisions depend on two countervailing forces: the centrifugal competition effect and the centripetal cost-saving effect. The focus of this paper is on the impact of vertical differentiation on the firms' location decisions based on their influence on the competition effect. We have argued that the higher the degree of vertical differentiation, the weaker will be the competition effect. This weakens the centrifugal competition effect leading to the emergence of the principle of minimum differentiation. Moreover, firms will not engage in price undercutting, since firms will agglomerate at the same site due to the heterogeneity of consumers' tastes in terms of quality. Several striking results appear as follows.

First of all, assuming that firms engage in discriminatory pricing in the commodity market, firms will agglomerate in the market's centre when the degree of vertical differentiation is sufficiently high, while they will move apart when it lies in between $\Delta_3^D \leq (\alpha_2 - \alpha_1) \leq \Delta_3^A$. Moreover, firms will locate further apart in the dispersal equilibrium when the degree of vertical differentiation between products is lower or the transport rate is higher. Despite all of this, the principle of maximum differentiation can never apply.

Second, firms agglomerate at the centre of the horizontal line segment as long as the degree of vertical differentiation is greater than zero, when firms engage in both uniform delivered and mill pricing. The reason why the dispersal equilibrium fails to exist in these two cases is that firms charge the same price at every point along the horizontal line leading to the result that the competition effect is much weakened.

Finally, we show that the principle of minimum differentiation may apply while the principle of maximum differentiation will never apply, even if the quality and location are endogenously determined. Moreover, we show that the min-max result may occur if the upper bound value of the quality level is not sufficiently high. However, the max-min result can never occur.

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Appendix

Substituting (13.1) into Equations (11.1)–(11.3), (12.1) and (12.2), we have:

$$\partial^2\Pi_1/\partial x_1^2|_{x_1^A,x_2^A} \le 0, \text{ if } (\alpha_2-\alpha_1) \ge \{t/[2(2-\bar{\theta})]\} \equiv \Delta_1^A, \tag{A.1}$$

$$\partial^2\Pi_2/\partial x_2^2|_{x_1^A,x_2^A} \le 0, \text{ if } (\alpha_2-\alpha_1) \ge \{t/[2(1+\bar{\theta})]\} \equiv \Delta_2^A, \tag{A.2}$$

$$J|_{x_1^A,x_2^A} \ge 0, \text{ if } (\alpha_2-\alpha_1) \ge \{3t/[2(2-\bar{\theta})(1+\bar{\theta})]\} \equiv \Delta_3^A, \tag{A.3}$$

$$q_1(1;\, x_2 = x_1 = 1/2) = (2-\bar{\theta})/3 > 0, \tag{A.4}$$

$$q_2(0;\, x_2 = x_1 = 1/2) = (1+\bar{\theta})/3 > 0. \tag{A.5}$$

Recall that $(2-\bar{\theta}) > 0$. We see from (A.1)–(A.3) that $\Delta_3^A > \Delta_1^A > \Delta_2^A$. Thus, central agglomeration emerges only if $(\alpha_2 - \alpha_1) \leq \Delta_3^A$.

Next, by substituting (13.2) into (11.1)–(11.3), (12.1) and (12.2), we have:

$$\partial^2\Pi_1/\partial x_1^2|_{x_1^D, x_2^D} \leq 0, \text{ if } (\alpha_2 - \alpha_1) \leq \{3t/[2(2-\bar{\theta})(2\bar{\theta}-1)]\} \equiv \Delta_1^D, \tag{A.6}$$

$$\partial^2\Pi_2/\partial x_2^2|_{x_1^D, x_2^D} = (2t)[-2(\alpha_2 - \alpha_1)(1+\bar{\theta})(2\bar{\theta}-1) - 5t]/[27(\alpha_2 - \alpha_1)] \leq 0, \tag{A.7}$$

$$J|_{x_1^D, x_2^D} \geq 0, \text{ if } (\alpha_2 - \alpha_1) \leq \{3t/[2(2-\bar{\theta})(1+\bar{\theta})]\} \equiv \Delta_2^D \equiv \Delta_3^A, \tag{A.8}$$

$$q_1(1;\, x_1^D, x_2^D) > 0, \text{ if } (\alpha_2 - \alpha_1) > \{4t/[(2-\bar{\theta})(5+2\bar{\theta})]\} \equiv \Delta_3^D, \tag{A.9}$$

$$q_2(0;\, x_1^D, x_2^D) > 0, \text{ if } (\alpha_2 - \alpha_1) > \{4t/[(1+\bar{\theta})(7-2\bar{\theta})]\} \equiv \Delta_4^D. \tag{A.10}$$

We find from (A.6) and (A.8) that $\Delta_2^D < \Delta_1^D$, from (A.9) and (A.10) that $\Delta_3^D > \Delta_4^D$, and from (A.3) and (A.8) that $\Delta_3^A = \Delta_2^D$. Accordingly, spatial dispersion occurs if $\Delta_3^D \leq (\alpha_2 - \alpha_1) \leq \Delta_3^A$.