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ENDOGENOUS QUALITY CHOICE: PRICE vs. QUANTITY COMPETITION*

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Two versions of a vertical product differentiation model, one with fixed and the other with variable costs of quality, are analysed to study how the hypotheses of price versus quantity competition affect equilibrium solutions. Product differentiation arises under all the scenarios considered, contrasting previous findings of symmetric quality choices under Cournot behaviour. However, to relax harsher market competition, firms differentiate more under Bertrand than under Cournot. A simple welfare measure also indicates that the economy is better off when firms compete on prices (with fixed costs of quality, not only consumer but also producer surplus is higher under price competition).

I. INTRODUCTION

WE ANALYSE a vertical product differentiation model with the aim of comparing (endogenous) equilibrium qualities under price and quantity competition. Under the simplifying assumptions that only two firms operate in the market, the following two-stage game is considered. At the first stage, firms choose the quality of the good they want to produce. At the second stage, a competitive process occurs where firms choose either prices or quantities. These two alternative cases are analysed and compared.

Two different assumptions are made about the nature of costs. In the first part of the paper, we assume that there are fixed costs of quality improvement, while variable costs do not change with quality. This may be thought of as a situation where firms should engage in R&D and advertising activities to improve quality. In the second part, we examine the case of variable costs of quality improvement, in the absence of fixed costs. This happens when the main burden of quality improvement falls, for instance, on more skilled labour or more expensive raw materials and inputs.

These two assumptions about costs correspond to the two canonical cases

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in the literature on vertical product differentiation. The former case has been analysed by Shaked and Sutton [1982, 1983, 1984] and Bonanno [1986], while the latter has been studied by Mussa and Rosen [1978], Gal-Or [1983] and Champsaur and Rochet [1989]. It is worth remembering that the properties of the model change dramatically according to these different assumptions on costs: the 'natural oligopoly' (or 'finiteness') property, i.e. the result that the number of firms which can coexist at equilibrium is bounded above, holds only when variable costs do not increase too steeply with quality, and can therefore be found only under the former assumption.

One of the main results of the paper is that firms always choose to offer distinct qualities at equilibrium, independently of the hypotheses on costs and on price vs. quantity competition. If this result is not surprising with respect to price competition, it is less expected for the case where firms compete on quantities. Actually, previous work incorporating this feature, both under fixed costs (Bonanno [1986], Ireland [1987]) and variable costs of quality (Gal-Or [1983]) finds a minimum differentiation solution.

We shall also see that the equilibrium qualities arising under price competition are more differentiated than those arising under quantity competition. This is because in the former case firms have a higher incentive to choose more distant specifications of the good, due to fiercer competition at the marketing stage of the game. This result is also robust across the two alternative specifications on costs of quality.

Finally, we shall prove through a simple welfare measure that the economy as a whole is better off under price competition than under quantity competition. This result is also robust, in the sense that it does not change with the different cost hypotheses. However, it must be anticipated that the result is much stronger when fixed costs are assumed, since in this case not only the consumer surplus but also the total profits in the industry are higher under Bertrand than under Cournot competition.

The plan of the paper is as follows. In the next section we present the basic model and deal with the case of fixed costs of quality, by analysing equilibria under both price and quantity competition and discussing the results. In section III we derive the Bertrand and Cournot solutions for the case of variable costs of quality, and check the robustness of the results obtained in the previous section. Section IV concludes the paper.

II. FIXED COSTS OF QUALITY IMPROVEMENT

In this section we present the basic model and we assume that there are fixed costs of quality. In section II(i) we analyse the case of price competition at the last stage of the game, while in section II(ii) the case of quantity competition is dealt with. Section II(iii) provides a comparison and a discussion of the results. The reader may find it useful to know the results of the analysis before going through the details. By defining $\bar{v}$ as the consumer endowed with the highest taste for quality in the economy, we can then anticipate the following propositions, proved in sections II(i) and II(ii) respectively.

Proposition 1. Under the assumption of *fixed costs* of quality improvement, the equilibrium of the game in which duopolists first choose qualities and then *prices* is such that a firm will select a quality $u_H^* = (0.2533)\,\bar{\upsilon}^2$, and the other the quality $u_L^* = (0.0482)\,\bar{\upsilon}^2$.

Proposition 2. Under the assumption of *fixed costs* of quality improvement, the equilibrium of the game in which duopolists first choose qualities and then *quantities* is such that a firm will select a quality $u_H^* = (0.2519)\,\bar{\upsilon}^2$, and the other the quality $u_L^* = (0.0902)\,\bar{\upsilon}^2$.

II(i). *Bertrand competition at the last stage*

In this section we introduce the model and derive the equilibrium solutions for the case in which firms choose first qualities simultaneously and then compete à la Bertrand at the market stage of the game. Consumers have the same (indirect) utility function $U = \upsilon u - p$ (and zero utility if they do not buy the differentiated good). They differ in their tastes, described by the parameter $\upsilon \in [\underline{\upsilon}, \bar{\upsilon}]$, υ being uniformly distributed with unit density. The higher the quality u of the good, the higher the utility U reached by the consumers for any given price p. However, consumers with a higher υ will be willing to pay more for a higher quality good. In accordance with the literature on product differentiation, we assume that consumers can buy at most one unit of the good.

Note that υ can be interpreted as the marginal rate of substitution between income and quality, so that a higher υ corresponds to a lower marginal utility of income and therefore a higher income (Tirole [1988, p. 96]). Under this interpretation, the model proposed here is the analog of the models where consumers differ by their incomes rather than by their tastes (Gabszewicz and Thisse [1979, 1980], Shaked and Sutton [1982, 1983, 1984], Bonanno [1986], Ireland [1987]).

We assume that there are only two firms in the industry, and that they play the following two-stage game. In the first stage, they decide on the quality u to be produced, with $u \geqslant 1$. There is no *a priori* upper bound to the level of quality, but we assume that there exists a lower bound to it. The latter can be interpreted as a minimum standard legal requirement or as being inherent to the production process. At this stage of the game, each firm incurs a fixed cost of the form $F_i = (u_i^2)/2$. In the second stage of the game, firms set prices. At this stage, costs of quality development have been already sunk and constant unit production costs are incurred. Without loss of generality, we take these costs to be zero. We look for the sub-game perfect Nash equilibrium of the game. As usual, this will be obtained by backward induction.

We first have to derive the equilibrium for the price-setting sub-game, taking as fixed qualities u_1 and u_2, with $u_1 \geqslant u_2$. The consumer indifferent between buying good 1 or good 2 has a taste parameter υ_{12} such that $\upsilon_{12} = (p_1 - p_2)/(u_1 - u_2)$. The consumer indifferent between buying the differentiated good and not buying at all has the taste parameter $\upsilon_{\emptyset 2} = p_2/u_2$. For this consumer, the purchase of the good of quality u_2 will imply zero utility level. The demand functions can then be easily built, noting that all the consumers for whom $\bar{\upsilon} \geqslant \upsilon \geqslant \upsilon_{12}$ will buy quality u_1, all those described by $\upsilon_{12} > \upsilon \geqslant \upsilon_{\emptyset 2}$ will buy quality u_2 and those described by $\upsilon_{\emptyset 2} > \upsilon$ will not buy at all. Notice that we assume that the market is not covered, i.e. some consumers do not buy the differentiated good ($\underline{\upsilon} < \upsilon_{\emptyset 2}$). We have chosen this formulation rather than that of full market coverage (often used in this literature for its simplicity), because under the latter the demand functions cannot be inverted and hence the Cournot case cannot be analysed.[1]

Quantity demanded to the high and low quality firm are given respectively by:

(1) $$q_1 = \bar{\upsilon} - (p_1 - p_2)/(u_1 - u_2)$$

(1′) $$q_2 = (p_1 - p_2)/(u_1 - u_2) - (p_2/u_2)$$

Firms choose prices to maximise their profits $\prod_i = p_i q_i$, for any given quality pair (u_1, u_2). We have the following first-order conditions:

(2) $$\partial \prod_1/\partial p_1 = \bar{\upsilon} + (p_2 - 2p_1)/(u_1 - u_2) = 0$$

(2′) $$\partial \prod_2/\partial p_2 = (p_1 - 2p_2)/(u_1 - u_2) - 2p_2/u_2 = 0$$

From these conditions we can derive the equilibrium prices charged by the high and the low quality firm respectively, as follows:

(3) $$p_1 = 2\bar{\upsilon} u_1 (u_1 - u_2)/(4u_1 - u_2)$$

(3′) $$p_2 = \bar{\upsilon} u_2 (u_1 - u_2)/(4u_1 - u_2)$$

The corresponding profits are:

(4) $$\prod_1(u_1, u_2) = 4(u_1 - u_2)\,[\bar{\upsilon} u_1/(4u_1 - u_2)]^2$$

(4′) $$\prod_2(u_1, u_2) = u_1 u_2 (u_1 - u_2)\,[\bar{\upsilon}/(4u_1 - u_2)]^2$$

We now look for the solutions of the quality game. Firms will choose their quality specification to maximise their profits $\pi_i = \prod_i(u_i, u_j) - (u_i^2)/2$ (for $i = 1, 2, i \neq j$), where $\prod_i(u_i, u_j)$ is equal to $\prod_1(u_i, u_j)$ if $u_i \geqslant u_j$, while it is equal to $\prod_2(u_i, u_j)$ if $u_i \leqslant u_j$. The first order conditions of this problem are:

(5) $$\partial \pi_1/\partial u_1 = 4u_1 \bar{\upsilon}^2 (4u_1^2 - 3u_1 u_2 + 2u_2^2)/[(4u_1 - u_2)^3] - u_1 = 0$$

(5′) $$\partial \pi_2/\partial u_2 = \bar{\upsilon}^2 u_1^2 (4u_1 - 7u_2)/[(4u_1 - u_2)^3] - u_2 = 0$$

[1] Under the assumption that the market is covered, we have that $q_1 + q_2 = \bar{\upsilon} - \underline{\upsilon}$. Since total demand $q_1 + q_2$ is not a function of p_1 and p_2, we are not able to find the inverse demand functions. Note that if we assumed $\underline{\upsilon} = 0$, then we would have that some consumers will never buy the good, and we would not need to make the explicit assumption that the market is not covered.

Now, rewrite (5) and (5′) by bringing u_1 and u_2 on the right-hand side of their respective equalities. After substituting and re-arranging, one obtains:

$$(6) \qquad 8u_2^3 - 12u_2^2u_1 + 23u_2u_1^2 - 4u_1^3 = 0$$

Set $u_1 = \mu u_2$, with $\mu \geqslant 1$ (recall that u_1 is by construction the higher quality, which allows us to do this transformation), so that (6) can be rewritten as:

$$(7) \qquad 4\mu^3 - 23\mu^2 + 12\mu - 8 = 0$$

The only solution (in the real numbers and greater than one) is $\mu = 5.2512$. By substituting this value back into the first-order conditions and using the relationship $u_1 = \mu u_2$, we obtain:

$$(8) \qquad u_1 = u_H^* = (0.2533)\bar{v}^2; \quad u_2 = u_L^* = (0.0482)\bar{v}^2$$

Expression (8) gives the pair of candidate equilibrium qualities. The second derivatives are

$$\partial^2\pi_1/\partial u_1^2 = -1 - 8u_2^2\bar{v}^2(5u_1 + u_2)/[(4u_1 - u_2)^4] \quad \text{and}$$

$$\partial^2\pi_2/\partial u_2^2 = -1 - 2u_1^2\bar{v}^2(8u_1 + 7u_2)/[(4u_1 - u_2)^4]$$

which are both negative, given that $u_1 \geqslant u_2$. This shows that (8) represent a local maximum. However, this is not enough to ensure we have found a Nash equilibrium. Let us imagine that firm i is producing the high quality u_1 and firm j is producing the low quality u_2. To be sure that our candidate maximum is indeed an equilibrium we also have to check that firm j has no incentive to "leapfrog" the rival firm and itself produce the highest quality. In other words, we have to prove that producing $u_2 = u_L^*$ in (8) is the optimal reply "from below" to the choice by the rival firm i to produce $u_1 = u_H^*$ also given by (8). Likewise, we have to prove that firm i has no incentive to deviate and produce a quality lower than that produced by firm j; i.e. we have to prove that producing u_H^* is the optimal reply "from above" to u_L^* along the whole spectrum of possible qualities. Formally, the following conditions have to be satisfied:

$$(9) \qquad \pi_j(u_L^*, u_H^*) \geqslant \pi_j(u_2, u_i = u_H^*) \quad \text{for } u_2 \leqslant u_H^*, \quad \text{and}$$

$$\pi_j(u_L^*, u_H^*) \geqslant \pi_j(u_1, u_i = u_H^*) \quad \text{for } u_1 \geqslant u_H^*.$$

$$(9') \qquad \pi_i(u_H^*, u_L^*) \geqslant \pi_i(u_1, u_j = u_L^*) \quad \text{for } u_1 \geqslant u_L^*, \quad \text{and}$$

$$\pi_i(u_H^*, u_L^*) \geqslant \pi_i(u_2, u_j = u_L^*) \quad \text{for } u_2 \leqslant u_L^*.$$

Expression (9) gives the conditions for the optimal reply of the firm producing the low quality, while expression (9′) gives the conditions for the optimal reply of the firm producing the high quality. Note that the second condition of (9) ensures that the low firm has no incentive to "leapfrog" its rival producing the highest quality itself, whereas the second expression of (9′) ensures that the high quality firm does not want to offer a lower quality than its low-quality rival.

Appendix I contains the proof that the candidate solution above is indeed a Nash equilibrium, i.e. satisfies conditions (9)-(9′).

As a final remark, we can derive the positions of the indifferent consumers, that is $\upsilon_{\emptyset 2} = (0.2116)\,\bar{\upsilon}$ and $\upsilon_{12} = (0.4754)\,\bar{\upsilon}$. Therefore, the analysis made so far requires the smallest taste parameter $\underline{\upsilon}$ in the distribution to be such that $\underline{\upsilon} < (0.2116)\,\bar{\upsilon}$ (otherwise, the assumption that the market is not covered would not hold).

The result we have obtained is not surprising. It is well known that firms choose to differentiate their products even when costs of quality are zero. This is because product differentiation allows firms to relax price competition on the market (Shaked and Sutton [1982]). However, it is worth relating our results with the main findings of the literature in vertical product differentiation, to point out some minor differences.

In Shaked and Sutton [1982] firms incur neither variable nor fixed costs of quality, and they can pick up their products within an interval $u_{min} \leqslant u \leqslant u_{max}$. At equilibrium, one firm chooses the maximum quality u_{max}, while the other selects a lower quality, which is strictly higher than u_{min}. Once fixed costs of quality are assumed (with variable costs remaining constant), then both qualities chosen at equilibrium will generally be internal to the interval of possible qualities. This is shown in Shaked and Sutton [1984], and it is exactly the result obtained in the present model.

Finally, note that like Shaked and Sutton [1982, 1984] we have found that at equilibrium the low quality firm offers a good which is higher than the minimum possible quality (for a high enough $\bar{\upsilon}$), whereas under the assumption that the market is covered it is optimal for one of the two firms to choose the lowest quality however high the taste parameter $\bar{\upsilon}$ (see for example Tirole [1988]; see also Choi and Shin [1992]).

II(ii). *Cournot competition*

In this section we derive the equilibrium solutions for the case where firms choose quantities rather than prices at the last stage of the game. To do so, we have to invert the system of demand functions given by (1) and (1′). This gives:

(10) $$p_1 = \bar{\upsilon}u_1 - q_2u_2 - q_1u_1$$

(10′) $$p_2 = (\bar{\upsilon} - q_1 - q_2)u_2$$

Firms choose quantities to maximise their profits, for any given quality pair (u_1, u_2). First-order conditions are:

(11) $$\partial\textstyle\prod_1/\partial q_1 = \bar{\upsilon}u_1 - q_2u_2 - 2q_1u_1 = 0$$

(11′) $$\partial\textstyle\prod_2/\partial q_2 = (\bar{\upsilon} - q_1 - 2q_2)u_2 = 0$$

Hence the optimal quantities produced by the high and the low quality firm are respectively:

(12) $q_1 = \bar{v} - (2u_1 - u_2)/(4u_1 - u_2)$

(12′) $q_2 = \bar{v}u_1/(4u_1 - u_2)$

Prices charged by the two firms are:

(13) $p_1 = \bar{v}u_1(2u_1 - u_2)/(4u_1 - u_2)$

(13′) $p_2 = \bar{v}u_1u_2/(4u_1 - u_2)$

Finally, the corresponding profits are given by:

(14) $\prod_1 = [\bar{v}(2u_1 - u_2)/(4u_1 - u_2)]^2 u_1$

(14′) $\prod_2 = [\bar{v}u_1/(4u_1 - u_2)]^2 u_2$

We now look for the solutions of the quality game. Firms will choose their quality specifications to maximise their profits $\pi_i = \prod_i(u_i, u_j) - (u_i^2)/2$ (for $i = 1, 2, i \neq j$). The first order conditions of this problem are:

(15) $\partial\pi_1/\partial u_1 = \bar{v}^2(2u_1 - u_2)(8u_1^2 - 2u_1u_2 + u_2^2)/[(4u_1 - u_2)^3] - u_1 = 0$

(15′) $\partial\pi_2/\partial u_2 = \bar{v}^2 u_1^2(4u_1 + u_2)/[(4u_1 - u_2)^3] - u_2 = 0$

To find the solution of the system given by (15)-(15′), set $u_1 = ku_2$, with $k \geqslant 1$, and re-arrange as follows:

(16) $\bar{v}^2(2k - 1)(8k^2 - 2k + 1) = ku_2(4k - 1)^3$

(16′) $\bar{v}^2 k^2(4k + 1) = u_2(4k - 1)^3$

Dividing (16) by (16′) and re-arranging, we obtain the following equation:

(17) $4k^4 - 15k^3 + 12k^2 - 4k + 1 = 0$

The only feasible solution is $k = 2.79243$. By substituting this value into the first-order conditions and using the relationship $u_1 = ku_2$, we obtain:

(18) $u_1 = .u_{H*} = (0.2519)\bar{v}^2$; $u_2 = .u_{L*} = (0.0902)\bar{v}^2$

This is the pair of candidate equilibrium solutions. The second derivatives are

$$\partial^2\pi_1/\partial_1^2 = -1 - 8u_2^2\bar{v}^2(u_1 - u_2)/[(4u_1 - u_2)^4] \quad \text{and}$$

$$\partial^2\pi_2/\partial u_2^2 = -1 + 2u_1^2\bar{v}^2(8u_1 - u_2)/[(4u_1 - u_2)^4],$$

which are negative when computed in the point given by (18). As in the previous case, to be sure that our candidate maximum is indeed an equilibrium we also have to check that the firm producing the lower (higher) quality has no incentive to "leapfrog" the rival and produce the higher (lower) quality itself. In other words, we have to check that conditions (9) and (9′)

hold. This is what we do in appendix II, which contains a proof that the two firms will not deviate from the pair (u_H^*, u_L^*). Proposition 2 anticipated at the beginning of section II then follows.

A comparison with earlier results on Cournot competition

The existing contributions on vertical product differentiation with quantity competition tend to point to a result of minimum differentiation. Bonanno [1986] proves that firms choose not to differentiate their qualities when fixed costs of quality improvement do not exist (see also Ireland [1987, pp. 71–4]) and gives a sufficient condition for which the same result holds in the presence of fixed costs.[2]

On the other hand, Bonanno [1986] builds a numerical example where he shows that maximum (not minimum) product differentiation may arise at equilibrium.

We want here to reconcile these former results with the outcome found above. We have proved that when there is no upper bound to quality choice and when fixed costs of quality exist, Cournot competition brings about product differentiation which is neither maximum nor minimum. We are now going to show that both these extreme results can be obtained within the same model once particular restrictions on the quality space are imposed.

Consider first the result of minimum differentiation. This is obtained by Bonanno [1986] under the following assumptions (see also Ireland [1987]): (a) costs are either zero or "do not increase too much" with quality; (b) there is an exogenous upper bound to the quality firms can select. By adding these two restrictions to our model we can find the same result. Consider the expressions (15) and (15′). They give the first derivatives of profits with respect to quality, for each firm. The first term at the left-hand side of each expression gives the marginal revenue of quality, while the last term represents the marginal cost of quality. It is easy to see that the former is always positive (recall that $u_1 > u_2$ by construction). If there are no costs of quality, this implies that firms will like to endlessly increase the quality they provide. If an exogenous upper constraint $u_{\max}$ is imposed, both firms will end up with choosing this highest quality independently of the rival's choice. This explains Bonanno's and Ireland's result of minimum differentiation when costs are zero.

More generally, in order for a result of no differentiation to be found, it is enough to introduce an upper bound to quality choice and assume that, when computed at this highest quality level $u_{\max}$, marginal costs are not as high as marginal revenues of quality. Formally, we must have $\partial \pi_i / \partial u_i \; (u_i = u_{\max}) > 0$ or equivalently: $\partial \prod_i / \partial u_i \; (u_i = u_{\max}) > \partial F_i / \partial u_i \; (u_i = u_{\max})$, for $i = 1, 2$. Given

[2] Firms also choose the same quality at equilibrium in a slightly different class of vertical product differentiation models with Cournot competition, where consumers are identical but are not constrained to buy only one unit of the good, unlike the present paper and the works cited so far (see Sutton [1991], Motta [1992a, 1992b]).

that $\partial\prod_i/\partial u_i$ is always positive, this amounts to saying that fixed costs F are either zero or do not increase too much with quality.

Consider next the result of maximum differentiation. Again, this can be derived from the model presented here once one introduces an appropriate constraint on the qualities that firms can choose. We know from the general case that firms want to differentiate their product even when they compete on quantities at the market stage of the game. However, if the interval of qualities they can pick up is sufficiently narrow, they cannot differentiate their products as much as they wish to. The quality constraint would be binding and firms will choose the quality values at the extreme of the admissible interval. To see this, consider again (15) and (15′). Let $u_{\min} = 1$ and $u_{\max} = 1.1$. By computing the derivatives at the point where firms select the extremes, one finds that $\partial\pi_2/\partial u_2(u_1 = 1, u_2 = 1.1) < 0$ and that $\partial\pi_2/\partial u_2(u_1 = 1,\ u_2 = 1.1) > 0$ (for $\bar{v} > 1.66$). Firms would like to put more distance between their product specifications, but given the exogenous constraint on their quality choice they end up with the corner solution where quality levels are located at the extremes of the interval $1 \leqslant u \leqslant 1.1$.

II(iii). *Bertrand vs. Cournot*

As to the comparison with the results arising under the Bertrand hypothesis, we note that Cournot competition will give rise to less product differentiation at equilibrium (see also Table I). This is a rather intuitive result:[3] since Cournot competition is less intense than under Bertrand competition (given the same qualities), firms still tend to separate from each other, but not as much as under Bertrand competition. Note, in particular, that in the Bertrand case a higher number of consumers is served, since the bottom quality firm tends to cover more of the bottom segment of the market ($v_{\varnothing 2}$ is lower). Further, under Bertrand behaviour the top quality on offer is higher. Along with lower prices charged by the firms, these features make competition on prices more beneficial to consumers than competition on quantities, as we can see from Table I, where consumer surpluses are also compared.[4]

[3] Contrast with Bonanno [1986, p. 83], who remarks that Cournot does not necessarily entail less product differentiation than Bertrand. This claim is based on the fact that extreme differentiation may arise under Cournot. The problem is that the Bertrand solution would also be one of extreme differentiation under the same assumptions on the quality interval. But once the constraints are no longer binding, one finds that Bertrand leads to higher product differentiation, as shown in the text.

[4] Consumer surplus has been computed using the following formula:

$$\int_{v_{\varnothing 2}}^{v_{12}} (vu_2 - p_2)\,dv + \int_{v_{12}}^{\bar{v}} (vu_1 - p_1)\,dv$$
$$= (\bar{v}^2/2)u_1 - p_1\bar{v} - [(v_{12})^2/2](u_1 - u_2) + v_{12}(p_1 - p_2) - (v_{\varnothing 2}{}^2/2)u_2 + p_2 v_{\varnothing 2},$$

where $v_{\varnothing 2}$ represents the consumer indifferent between purchasing the differentiated good or not, and v_{12} represents the consumer indifferent between buying good 1 or good 2.

TABLE I

EQUILIBRIUM VALUES, UNDER THE DIFFERENT HYPOTHESES ON FIRMS' BEHAVIOUR, AND UNDER DIFFERENT ASSUMPTIONS ON COSTS OF QUALITY

	Fixed Costs of Quality Improvement					
	General Case			*Example:* $\bar{v} = 5$		
	Firm 1	*Firm 2*		*Firm 1*	*Firm 2*	
Bertrand competition	$u_1 = 0.2533\bar{v}^2$ $p_1 = 0.1077\bar{v}^3$ $q_1 = 0.5246\bar{v}$ $\pi_1 = 0.0244\bar{v}^4$	$u_2 = 0.0482\bar{v}^2$ $p_2 = 0.0102\bar{v}^3$ $q_2 = 0.2638\bar{v}$ $\pi_2 = 0.0015\bar{v}^4$	$v_{\varnothing 2} = 0.2116\bar{v}$ $v_{12} = 0.4754\bar{v}$ $CS = 0.0432\bar{v}^4$ $PS = 0.0259\bar{v}^4$	$u_1 = 6.33$ $p_1 = 13.46$ $q_1 = 2.62$ $\pi_1 = 15.26$	$u_2 = 1.2$ $p_2 = 1.27$ $q_2 = 1.32$ $\pi_2 = 0.96$	$v_{\varnothing 2} = 1.06$ $v_{12} = 2.38$ $CS = 27.0$ $PS = 16.22$
Cournot competition	$u_1 = 0.2519\bar{v}^2$ $p_1 = 0.1136\bar{v}^3$ $q_1 = 0.4508\bar{v}$ $\pi_1 = 0.0195\bar{v}^4$	$u_2 = 0.0902\bar{v}^2$ $p_2 = 0.0248\bar{v}^3$ $q_2 = 0.2747\bar{v}$ $\pi_2 = 0.0027\bar{v}^4$	$v_{\varnothing 2} = 0.2745\bar{v}$ $v_{12} = 0.5492\bar{v}$ $CS = 0.0402\bar{v}^4$ $PS = 0.0222\bar{v}^4$	$u_1 = 6.30$ $p_1 = 14.2$ $q_1 = 2.25$ $\pi_1 = 12.17$	$u_2 = 2.25$ $p_2 = 3.1$ $q_2 = 1.37$ $\pi_2 = 1.71$	$v_{\varnothing 2} = 1.37$ $v_{12} = 2.75$ $CS = 25.1$ $PS = 13.87$
	Variable Costs of Quality Improvement					
				Example: $\bar{v} = 5$		
				Firm 1	*Firm 2*	
Bertrand competition				$u_1 = 4.10$ $p_1 = 11.33$ $q_1 = 1.40$ $\pi_1 = 4.10$	$u_2 = 1.99$ $p_2 = 3.75$ $q_2 = 1.72$ $\pi_2 = 3.04$	$v_{\varnothing 2} = 1.88$ $v_{12} = 3.60$ $CS = 11.75$ $PS = 7.14$
Cournot competition				$u_1 = 3.69$ $p_1 = 10.84$ $q_1 = 1.09$ $\pi_1 = 4.41$	$u_2 = 2.93$ $p_2 = 7.86$ $q_2 = 1.22$ $\pi_2 = 4.37$	$v_{\varnothing 2} = 2.69$ $v_{12} = 3.91$ $CS = 8.30$ $PS = 8.78$

Legend: CS = consumer surplus; PS = producer surplus (sum of profits). See text for other notations.

If it is hardly surprising that consumers will benefit from harsher competition in the product market, far less obvious is the result that the producer surplus is higher under Bertrand competition, as shown in Table I. This outcome is clearly driven by the choice of quality made by the oligopolists: anticipating tougher competition at the last stage of the game, they select more distant specifications of the goods. This will go to the advantage of the top quality firm above all, but the total profits gained by the producers are nevertheless higher under Bertrand than under Cournot.

Our result contrasts with Vives [1985] who shows that profits are always higher under Cournot than under Bertrand. This opposite result is due to the fact that that author studies a non-spatial model of differentiation where product specifications are exogenously given.[5]

A twofold warning should be added to this discussion. Firstly, the result that profits are higher under price than under quantity competition has been derived in a model with a fixed number of firms. The outcome might be different in a free entry model. Secondly, this property is not robust across different hypotheses on costs. We shall see in the next section that when variable—instead of fixed—costs of quality are assumed, profits are higher under quantity than under price competition.

III. VARIABLE COSTS OF QUALITY IMPROVEMENT

In this section, we analyse the case where costs of quality improvement fall upon variable costs instead of fixed costs. A quadratic function will be assumed, but it will intervene in the unit costs. No fixed cost is assumed. Like in the previous section, the analysis deals with two different scenarios. In section III(i), firms choose first qualities and then prices. In section III(ii), firms choose first qualities and then quantities.[6]

The analysis carried out in this section will confirm the main results obtained in the case of fixed costs of quality. In particular, we shall show that firms always choose distinct qualities, and that more product differentiation

[5] Bertrand profits are also higher in Deneckere [1983] who considers supergames in a non-spatial model of product differentiation. Singh and Vives [1984] analyse a non-spatial model of a differentiated duopoly (where qualities are not endogenous) and consider the following two-stage game. In the first period, firms simultaneously commit themselves to a type of contract with customers, either on quantities or prices. In the second, they compete contingent on the chosen type of contract. If such a game was considered within our model, and a scheme to compensate the firm producing the low quality good was devised, then we would find that producers choose the price contract. Instead, Singh and Vives [1984] prove that the quantity contract is chosen as a dominant strategy by the firms whenever they provide–like in the case we analyse–substitute goods.

[6] Given that the role of fixed sunk costs does not appear any longer under the assumptions on costs made here, the results would not change if firms chose simultaneously qualities and prices (quantities). We retained the sequential formulation for symmetry with the case dealt with in section II.

occurs under price than under quantity competition. We anticipate here the propositions established in this section.

Proposition 1'. Under the assumption of *variable costs* of quality improvement, the equilibrium of the game in which duopolists first choose qualities and then *prices* is such a firm will select a quality $u_H^* = 4.10$, and the other the quality $u_L^* = 1.99$ (for $\bar{v} = 5$).

Proposition 2'. Under the assumption of *variable costs* of quality improvement, the equilibrium of the game in which duopolists first choose qualities and then *quantities* is such that a firm will select a quality $u_H^* = 3.69$, and the other the quality $u_L^* = 2.93$ (for $\bar{v} = 5$).

III(i). *Price competition*

The specification of the model presented in section II is maintained. The only difference is that $F(u) = u^2/2$ are now variable rather than fixed costs. This will be taken into account when computing profits. Quantities are still given by (1) and (1'). It is then easy to write firms' profits, compute first derivatives with respect to prices and solve the corresponding system of equations, which results in the following:

(19) $$p_1 = u_1(2u_1^2+4u_1\bar{v}-4u_2\bar{v}+u_2^2)/[2(4u_1-u_2)]$$

(19') $$p_2 = u_2(u_1^2+2u_1\bar{v}-2u_2\bar{v}+2u_1u_2)/[2(4u_1-u_2)]$$

Profits at the first stage of the game can then be showed to be:

(20) $$\pi_1 = u_1^2(u_1-u_2)(2u_1+u_2-4\bar{v})^2/[4(4u_1-u_2)^2]$$

(21') $$\pi_2 = u_1u_2(u_1-u_2)(u_1-u_2+2\bar{v})^2/[4(4u_1-u_2)^2]$$

The system of first-order conditions to be solved is then the following:

(22) $$\partial\pi_1/\partial u_1 = [u_1(2u_1+u_2-4\bar{v})(24u_1^3-22u_1^2u_2+5u_1u_2^2+2u_2^3 - 16u_1^2\bar{v}+12u_1u_2\bar{v}-8u_2^2\bar{v})]/[4(4u_1-u_2)^3] = 0$$

(22') $$\partial\pi_2/\partial u_2 = [u_1(u_1-u_2+2\bar{v})(4u_1^3-19u_1^2u_2+17u_1u_2^2-2u_2^3 + 8u_1^2\bar{v}-14u_1u_2\bar{v}-8u_2^2\bar{v})]/[4(4u_1-u_2)^3] = 0$$

To find an analytical solution to this system does not seem to be easy. However, we have been able to find a solution once we specify the value of $\bar{v}$. For instance, for $\bar{v} = 5$, it is possible to show that the unique pair of values which solves system (22)-(22') is given by:

(23) $$u_1 = u_H^* = 4.0976; \quad u_2 = u_L^* = 1.9936$$

The second derivatives of profits computed in this point are negative; further, the pair (23) represents a Nash equilibrium of the game since it respects conditions (9) and (9') stated above. A proof of this result, representing Proposition 1′ anticipated at the beginning of the present section III, can be found in appendix III below.

This result of product differentiation at equilibrium when firms compete in prices and incur variable costs of quality is in line with earlier work (e.g. Champsaur and Rochet [1989]).

III(i). *Quantity competition*

In this section, we continue to deal with the hypothesis that firms have variable costs of quality but price competition is replaced by quantity competition. A previous work in this setting is Gal-Or [1983], who generalises Mussa and Rosen [1978] to analyse the impact of entry in a vertical product differentiation model where qualities and quantities are endogeneously determined by the firms. The main difference with respect to the model we are going to present lies in the fact that Gal-Or assumes that firms choose a spectrum of products *and* decide on qualities and quantities simultaneously. Here we build a simple example which can be seen as a variant of Gal-Or's model, and we prove that the identical choice of quality corresponds not to the maximum, but to the *minimum* profits the firms can reach. Instead, the equilibrium solution arises with the duopolists choosing to differentiate their products at equilibrium. This will prove the robustness of the result already found in section II: even with Cournot competition, firms will select distinct qualities at equilibrium, independently of whether they incur fixed or variable costs of quality.

Intuitively, the scope for symmetric equilibria to arise under variable costs of quality is even weaker than with fixed costs. With the former, there is more opportunity for the firms to segment the market than with the latter. Under this hypothesis, firms can better exploit their potential "captive" market and have a higher incentive to separate from each other: firms with lower (higher) quality can produce it at a lower (higher) variable cost and sell it at a lower (higher) price to consumers having lower (higher) taste for quality. Instead, when quality costs are fixed and production costs are constant this biunivocal correspondence between variable costs, prices, qualities and consumer tastes does not hold any more.[7] Hence, the reader should not be surprised that product differentiation also arises under variable costs and quantity competition.

We maintain the specification used in section III(i) by keeping the assumption that the quality costs $F(u) = (u^2)/2$ are *variable* costs but assuming quantity competition in the second stage of the game. The demand and

[7] Sutton [1991, ch. 3] calls the case of variable costs increasing with quality "Chamberlinian" case of vertical product differentiation. He also underlines that this case is structurally similar to the horizontal product differentiation models.

inverse demand functions given by (1)-(1′) and (10)-(10′) still hold good. At the last stage of the game, the first-order condition will be:

(24) $$\partial\prod_1/\partial q_1 = \bar{v}u_1 - q_2u_2 - 2q_1u_1 - (u_1^2)/2 = 0$$

(24′) $$\partial\prod_2/\partial q_2 = (\bar{v} - q_1 - 2q_2)u_2 - (u_2^2)/2 = 0$$

The solution to this system of equations is given by:

(25) $$q_1 = (-2u_1^2 + u_2^2 + 4u_1\bar{v} - 2u_2\bar{v})/(8u_1 - 2u_2)$$

(25′) $$q_2 = (u_1^2 - 2u_1u_2 + 2u_1\bar{v})/(8u_1 - 2u_2)$$

At the first stage, firms choose qualities to maximise their profits (recall that unit production costs are a quadratic function of quality), given by:

(26) $$\pi_1 = u_1(2u_1^2 - u_2^2 - 4u_1\bar{v} + 2u_2\bar{v})^2/[4(4u_1 - u_2)^2]$$

(26′) $$\pi_2 = u_1^2u_2(u_1 - 2u_2 + 2\bar{v})^2/[4(4u_1 - u_2)^2]$$

The first-order conditions are:

(27) $$\partial\pi_1/\partial u_1 = [(2u_1^2 - u_2^2 - 4u_1\bar{v} + 2u_2\bar{v})(24u_1^3 - 10u_1^2u_2 + 4u_1u_2^2 + u_2^3 - 16u_1^2\bar{v} + 4u_1u_2\bar{v} - 2u_2^2\bar{v})]/[4(4u_1 - u_2)^3] = 0$$

(27′) $$\partial\pi_2/\partial u_2 = u_1^2(u_1 - 2u_2 + 2\bar{v})(4u_1^2 - 23u_1u_2 + 2u_2^2 + 8u_1\bar{v} + 2u_2\bar{v})/[4(4u_1 - u_2)^3] = 0$$

The symmetric solution to this system is given by $u_1 = u_2 = 2\bar{v}$. However, one can derive the second derivatives and check that $\partial^2\pi_1/\partial u_1^2 = \partial^2\pi_2/\partial u_2^2 = 4\bar{v}/9$ when computed in correspondence of the candidate maximum.[8] In other words, choosing this same quality would give the firms a minimum profit.[9]

We now turn to the question of whether a Nash equilibrium for this game exists at all. Dealing with the system given by (27)-(27′) is not analytically simple. It is possible to show that the values of u_1, u_2 which simultaneously satisfy the two conditions are those which set the last factor of the numerators of both (27) and (27′) equal to zero. By solving the resulting equations, it is possible to find the solution of the system. As an example, consider $\bar{v} = 5$. In this case, we look for the values u_1, u_2 which solve:

[8] We omit the expression of $\partial^2\pi_1/\partial u_1^2$ because of its length. As to the other derivative, it is given by:

$$\partial^2\pi_2/\partial u_2^2 = -[u_1^2(7u_1 - 2`u)(8u_1^2 - 23u_1u_2 + 16u_1`u + 2u_2`u)]/[2(4u_1 - u_2)^4].$$

From this the reader can verify that the second order conditions are not satisfied at the symmetric candidate equilibrium.

[9] Of course, symmetric equilibria may arise under functional forms other than those chosen here. I am grateful to Esther Gal-Or who, after this work was finished, brought my attention to two works of hers previously unknown to me where she also finds asymmetric qualities at equilibrium (Gal-Or [1985, 1987]). These two papers adopt a slightly different utility function but they share the same framework as that I use in this section.

(28) $$24u_1^3 - 10u_1^2u_2 + 4u_1u_2^2 + u_2^3 - 80u_1^2 + 20u_1u_2 - 10u_2^2 = 0$$

(28′) $$4u_1^2 - 23u_1u_2 + 2u_2^2 + 40u_1 + 10u_2 = 0$$

The reader can check that the following pair solves the system given by (28)-(28′):[10]

(29) $$u_1 = u_H^* = 3.69048, \quad u_2 = u_L^* = 2.92788$$

Further, the second derivatives computed at this candidate maximum are negative, being $\partial^2\pi_1/\partial u_1^2 = -1.04$ and $\partial^2\pi_2/\partial u_2{}^2 = -1.02$.

Appendix IV shows that this pair is a global maximum and that it gives the optimal replies both from below and from above in the sense of condition (9)-(9′), thus proving Proposition 2′ stated at the beginning of this section.

The simple example we have just analysed shows not only that the symmetric solution does not represent an equilibrium of the game with quantity competition but also that the equilibrium involves the choice of different products by the firms. This confirms, in a setting with variable costs increasing with quality, the result obtained in the preceding section II under the assumption of fixed costs increasing with quality.

III(iii). *Discussion*

In this section we have showed that product differentiation arises under both price and quantity competition. Like the case of fixed costs of quality, we have seen that earlier results of identical choice of quality when firms compete à la Cournot do not hold good.

Table 1 gives the equilibrium values corresponding to the cases dealt with in this section. We can see that price competition involves more differentiation than quantity competition, thus confirming an outcome found in section II. The intuition of this result is the same as the one provided in the past section: price competition involves "harsher" competition than when firms choose quantities. By anticipating this outcome, firms try to relax market competition by choosing qualities that are further apart from each other.

Unlike the case of fixed costs, however, one can see from Table I that total profits are higher under quantity than under price competition. The difference does not seem to have a straightforward intuition.[11]

[10] Numerical computations have been performed using the programme "Mathematica". The exact solutions have respectively 18 and 16 decimals.

[11] It probably depends on the fact that in the variable cost case, a biunivocal relationship is established under quality, variable costs and prices. This implies that an increase in quality tends to give less profits than under the assumption of fixed costs. Recall that in that case, under Bertrand competition, the increase in profits comes from the firm which is pushed towards a higher quality. When firms incur variable costs of quality, the increase in quality which occurs under price (with respect to quantity) competition does not give the same profit advantage to the top quality firm.

The reader can also check from Table I that a welfare measure consisting of the sum of consumer and producer surpluses reveals that welfare is higher under price than under quantity competition. In other words, gains by consumers outweigh losses by firms (in the case of fixed costs, both consumers and producers as a whole were better off with price competition).

Finally, it is worth adding that the analysis carried out in section III is based on particular values of $\bar{v}$. However, it can be easily shown that the nature of the results identified above and summarised in Table I does not change when $\bar{v}$ varies.

IV. SUMMARY AND CONCLUSIONS

In this paper we have analysed a vertical differentiation model to compare how different hypotheses at the market stage of the game affect equilibrium qualities, prices and profits. Among other things, we have proved that *product differentiation always arises at equilibrium.* For the case of quantity competition, this finding contradicts previous results obtained in the literature.

We have also found that under the hypothesis of price competition firms will differentiate their product specifications more than under quantity competition. It is the anticipation of fiercer competition at the last stage of the game which pushes firms to choose more differentiated products than under Cournot competition.

Finally, we have seen that welfare is higher when firms compete in prices rather than in quantities.

The results briefly summarised here hold under the two different cases of fixed and variable costs of quality. Of course, they have been found by relying on a very simple model. Among other things, their robustness should also be checked across different functional forms for consumer preferences and costs. The assumption of a fixed number of firms should also be relaxed, to see if the results hold good once free entry is allowed.[12]

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[12] Note however that under the assumptions of (quadratic) fixed costs of quality the finiteness property would hold good in the cases of both price and quantity competition at the last stage of the game (Shaked and Sutton [1987]). This suggests that, for the case studied in section II, it might be possible to find conditions for which only two firms would operate at equilibrium under both hypotheses even if free entry were allowed. In contrast, the finiteness property would not hold under the assumption of variable costs made in section III.

APPENDIX I

To prove that (8) represents a Nash equilibrium according to the definition stated in (9) and (9′), we show (a) that the low quality firm has no incentive to deviate from u_L^* and to "leapfrog" the rival when the latter produces u_H^* given by (8), and (b) that the high quality firm has no incentive to deviate from u_H^* by producing a quality whose level is lower than u_L^* offered by its rival.

(a) By producing a quality level u_1 higher than u_H^*, the low quality firm would earn: $\pi_1(u_1 | u_H^*) = 4(u_1 - u_H^*)[\bar{v}u_1/(4u_1 - u_H^*)]^2 - (u_1^2)/2$. Letting $u_1 = ku_H^*$ with $k \geqslant 1$, and substituting $u_H^* = (0.2533)\bar{v}^2$, we have: $\pi_1 = (0.2533)k^2\bar{v}^2[(0.2533) \times (k-1)\bar{v}^2 - (0.2533)\bar{v}^2(4k-1)^2]/[2(4k-1)^2]$. It can readily be verified that the expression in square brackets at the numerator is negative for any value of $k \geqslant 1$, which leads to the result: $\pi_1(u_1, u_H^*) < 0 < \pi_2(u_L^*, u_H^*)$. ■

(b) If the high quality firm decided to choose a quality lower than u_L^*, it would obtain a profit $\pi_2(u_2 | u_L^*) = u_L^* u_2(u_L^* - u_2)[\bar{v}/(4u_L^* - u_2)]^2 - (u_2^2)/2$. The function $u_L^* u_2(u_L^* - u_2)[\bar{v}/(4u_L^* u_2)]^2$ attains its maximum in $u_2 = 4u_L^*/7$, where it has the value $(0.0208)u_L^*$. By substituting the value of u_L^* given by (8), we can then write: $\pi_2(u_2, u_L^*) < (0.001)\bar{v}^4 < 0.0244\bar{v}^4 = \pi_1(u_H^*, u_L^*)$. ■

We have therefore proved that the firms will not deviate from the pair (u_H^*, u_L^*), thus also proving Proposition 1 stated at the beginning of section II.

APPENDIX II

To prove that (18) represents a Nash equilibrium according to the definition stated in (9) and (9′), we show (a) that the low quality firm has no incentive to deviate from u_L^* and "leapfrog" the rival when the latter produces u_H^* given by (18), and (b) that the high quality firm has no incentive to deviate from u_H^* by producing a quality whose level is lower than u_L^* offered by its rival.

(a) Producing a quality u_1 higher than u_H^* would give the low quality firm the profit

$$\begin{aligned}\pi_1(u_1 | u_H^*) &= u_1[\bar{v}(2u_1 - u_H^*)/(4u_1 - u_H^*)]^2 - (u_1^2)/2\\ &= u_1\{[\bar{v}^2(2u_1 - u_H^*)^2 - u_1(4u_1 - u_H^*)^2]/\\ &\quad [2(4u_1 - u_H^*)^2]\} < u_1\{[\bar{v}^2(2u_1 - u_H^*)^2 - u_1(4u_1 - 2u_H^*)^2]/\\ &\quad [2(4u_1 - u_H^*)^2]\} = u_1\{(\bar{v}^2 - 4u_1)(2u_1 - u_H^*)^2/[2(4u_1 - u_H^*)^2]\}.\end{aligned}$$

The last expression is bounded above by $u_1[(\bar{v}^2 - 4u_1)/8]$. Since the latter function has as its maximum value $\bar{v}^4/128$ (reached in the point $u_1 = \bar{v}^2/8$), we can write:

$$\pi_1(u_1 | u_H^*) < \bar{v}^4/128 < (0.0195)\bar{v}^4 = \pi_2(u_L^*, u_H^*) \quad ■$$

(b) If the high quality firm decided to choose a quality lower than u_L^*, it would obtain a profit

$$\pi_2(u_2 | u_L^*) = (u_L^*)^2 u_2[\bar{v}/(4u_L^* - u_2)]^2 - (u_2^2)/2.$$

The function $(u_L^*)^2 u_2[\bar{v}/(4u_L^* - u_2)]^2$ attains its maximum in $u_2 = u_L$, where its value is $\bar{v}^2 u_L^*/9$. By substituting the value of u_L^* given by (18), we can then write:

$$\pi_2(u_2, u_L^*) < (0.010)\bar{v}^4 < 0.0195\bar{v}^4 = \pi_1(u_H^*, u_L^*). \quad ■$$

We have therefore proved that the two firms will not deviate from the pair (u_H^*, u_L^*). Proposition 2 stated at the beginning of section II then follows.

APPENDIX III

To show that (23) is a Nash equilibrium in the sense of (9)-(9′), we show that (a) the low quality firm has no incentive to deviate from u_L^* to produce a higher quality than u_H^*, and (b) that the high quality firm has no incentive to deviate from u_H^* to produce a quality lower than its rival.

(a) By selling a quality u_1 higher than u_H^*, the low quality firm would earn a profit:

$$\pi_1(u_1 | u_H^*) = u_1^2(u_1 - u_H^*)(2u_1 + u_H^* - 20)^2/[4(4u_1 - u_H^*)^2].$$

The maximum of the profit function in the interval where outputs are positive (after $u_1 > 7.9$, the corresponding quantity becomes negative) is reached for $u_1 = 5.286$. But $\pi_1(u_1 = 5.286 | u_H^*) = 0.81 < \pi_2(u_L^*, u_H^*) = 3.03$. The low quality firm has therefore no incentive to leapfrog the rival. ■

(b) If the top quality firm decided to produce a quality lower than its rival, it would earn:

$$\pi_2(u_2 | u_L^*) = u_L^* u_2(u_L^* - u_2)(u_L^* - u_2 + 10)^2/[4(4u_L^* - u_2)^2],$$

where u_L^* is defined by (23). The maximum value reached by this profit function is $\pi_2(u_2 = 1.049 | u_L^*) = 1.23 < \pi_1(u_L^*, u_H^*) = 4.1$. This completes the proof that (23) is the Nash equilibrium of the game when $\bar{v} = 5$. ■

Proposition 1′ stated at the beginning of section III is then proved.

APPENDIX IV

We first prove (a) that the low firm has no incentive to leapfrog the high quality, and then (b) that the high quality firm has no incentive to select a quality lower than its rival.

(a) If the low quality firm decided to provide a quality higher than u_H^* and leapfrog its rival, it would obtain the following profits: $\pi_1(u_1 | u_H^*) = u_1(2u_1^2 - u_H^{*2} - 20u_1 + 10u_H^*)^2/[4(4u_1 - u_H^*)^2]$. This function is decreasing in u_1 in the admissible interval (the profit function makes sense only for the interval $1 \leqslant u_1 \leqslant 8.8$, because of the conditions on the positivity of outputs: the reader can check that (25) is non-negative when $\bar{v} = 5$ only in this interval). It is then possible to see that:

$$\pi_1(u_1 = 3.69 | u_H^*) = 4.08 < \pi_2(u_L^*, u_H^*) = 4.37. \quad ■$$

(b) If the top quality firm decided to deviate from (29) to produce a quality lower than its rival, it would earn: $\pi_2(u_2 | u_L^*) = u_L^{*2} u_2(u_L^* - 2u_2 + 10)^2/[4(4u_L^* - u_2)^2]$, where u_L^* is defined by (29). The maximum of this function (for positive outputs) is given by $\pi_2(u_2 = 2.93 | u_L^*) = 4.06 < \pi_1(u_L^*, u_H^*) = 4.40$. There is then no incentive for the top quality firm to deviate. ■

Proposition 2′ of section III is then proved.

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