The vertical differentiation model in the insurance market: costs structure and equilibria analysis

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Abstract. We investigate the vertical differentiation model in the insurance market taking into account fixed costs (the costs of quality improvement) of different structure. The subgame perfect equilibrium in a two-stage game is constructed for the case of compulsory insurance when firms use Bertrand-Nash or Stakelberg equilibria at the stage of price competition. For the case of optional insurance we explore and compare the firms optimal behavior in monopoly and duopoly settings.

Keywords: vertical differentiation model, insurance, equilibrium, quality, duopoly, monopoly pricing, fixed costs.

1. Introduction

The vertical differentiation is a useful strategy to avoid fierce price competition that can be used in the markets of different nature (see, for instance, Gabszewics and Thisse, 1979; Kuzyutin et al., 2007; Motta, 1993; Ronnen, 1991; Shaked and Sutton, 1982; Tirole, 1988; Zenkevich and Kuzyutin, 2006). Several vertical differentiation models in the insurance market were offered early in (Okura, 2010; Schlesinger and Schulenburg, 1993; Schlesinger and Schulenburg, 1991).

Our research is based partly on the assumptions and the vertical differentiation model, explored in (Okura, 2010). The main differences are:

- another interpretation of "average variable cost parameter" v which generates another variable cost function and profit function;
- we consider three different types of fixed costs (the costs of quality improvement): zero fixed costs, linear and quadratic fixed costs;
- we suppose the price equilibrium can be either Nash equilibrium or Stakelberg equilibrium;
- we try to find the firms optimal behavior in the case of compulsory (mandatory) and optional insurance.

Namely, we examine the "quality – price 2-firm competition model" (Zenkevich and Kuzyutin, 2006; Kuzyutin et al., 2007) in the insurance market using the well-known vertical differentiation framework (Tirole, 1988; Ronnen, 1991). The competition between insurance firms takes place in a two-stage game. In the first stage they decide on the quality (level of claims handling procedure) $q_i,\ i=1,2$, to offer (let $0 \le \underline{q} \le q_1 < q_2 \le \overline{q}$). At this stage each firm faces fixed costs $FC(q_i)$. In the second stage the firms choose simultaneously (the case of Bertrand-Nash equilibria) or sequentially (the case of Stakelberg equilibria) their prices $p_i, i=1,2$ (insurance rates). The solution concept traditionally used in similar models is a subgame perfect equilibrium (Selten, 1975) which can be constructed using the backwards induction procedure (see, for instance, (Petrosjan and Kuzyutin, 2008)).

In section 2 we set up the quality – price competition model of vertical differentiation in the insurance market (compulsory insurance case). The second-stage Nash price equilibrium is derived in section 3. In section 4 we find out the optimal firms behavior at the stage of quality competition depending on the fixed costs function structure: zero, linear or quadratic fixed costs. In section 5 we assume that firms engage the Stakelberg equilibrium at the stage of price competition. Both sections 6 and 7 are devoted to so called "optional insurance case". The optimal monopoly behavior is derived in section 6 (with and without vertical differentiation). In section 7 we managed to construct the subgame perfect equilibrium in the case of duopoly settings in the insurance market.

2. The vertical differentiation in the insurance market: compulsory insurance case

Suppose there are two insurance firms (firm 1 and firm 2) in the market, and both offer the compulsory insurance services with different quality (the level of claims handling procedure) $q_i \in \left[\underline{q}, \overline{q}\right],\ \ 0 \le \underline{q} < \overline{q} < +\infty$. Without loss of generality let us assume that firm 1 is a low-quality insurance firm and firm 2 is a high-quality insurance firm: $\underline{q} \le q_1 < q_2 \le \overline{q}$.

Let t be the accident probability for the consumer, and both firms assume the parameter t is uniformly distributed on the segment $\left[0, \overline{t}\right],\ \ \overline{t} \le 1$.

Each firm i tries to offer such contract (q_i, p_i), where p_i denotes the price (insurance rate) that maximizes her profit. We first assume that each consumer has to purchase one insurance service (product) from a more desirable insurance firm (the case of mandatory insurance). Each consumer's strategy is maximizing his utility function of the following form:

$$U_t = \max\left\{tq_1 - p_1, tq_2 - p_2\right\}. \tag{1}$$

We guess the firms decision making process is a two-stage game. In the first stage (the stage of quality competition) both firms simultaneously choose the qualities $q_i \in \left[\underline{q}, \overline{q}\right]$ of their insurance products and, thus, they face some quality development costs $FC(q_i)$. The first stage decisions q_1 and q_2 become observable before the second stage starts. At the second stage (the stage of price competition) the rivals charge their prices p_1 and p_2 respectively.

Then each consumer faces two substitute contracts (q_1, p_1) and (q_2, p_2), and consumer's reaction due to (1) determines the firms market shares D_1 and D_2.

The marginal consumer $\widehat{t}$ is indifferent between buying the insurance product from firm 1 and firm 2:

$$\hat{t} = \frac{p_2 - p_1}{q_2 - q_1}.$$

In the case of vertical differentiation $(0 < \widehat{t} < \overline{t})$ the firms market shares are determined as follows: $D_1 = \left[0, \hat{t}\,\right]$, $D_2 = \left[\hat{t},\ \overline{t}\,\right]$.

The expected profit functions of the insurance firms (in the case of compulsory insurance) can be written in the following form:

$$\begin{aligned} \mathit{\Pi}_1 &= \tfrac{1}{\bar{t}} \left[\hat{t} \cdot p_1 - \int\limits_0^{\hat{t}} v \cdot t dt \right] - FC(q_1) \\ \mathit{\Pi}_2 &= \tfrac{1}{\bar{t}} \left[(\bar{t} - \hat{t}) \cdot p_2 - \int\limits_{\hat{t}}^{\bar{t}} v \cdot t dt \right] - FC(q_2) \end{aligned} \tag{2}$$

where $v > 0$ represents the "average variable cost parameter", and $v \cdot t$ is assumed to be the expected cost of the claims handling procedure for the consumer t.

Let us denote the model of vertical differentiation in the insurance market described above by the QP-model (compulsory insurance case).

3. The second-stage (price) equilibrium

The solution concept traditionally used in similar models of vertical differentiation is a subgame perfect equilibrium (SPE). The equilibrium can be constructed by the backwards induction procedure.

Given q_1 and q_2 (the firms decisions at the stage of quality competition), $\underline{q} \le q_1 < q_2 \le \overline{q}$, one can find the second-stage price equilibrium.

Proposition 1. *In the QP-model (compulsory insurance case) there exists second-stage Nash price equilibrium*

$$\begin{cases} p_1^{NE} = \dfrac{\bar{t}}{3}\big(v + q_2 - q_1\big) \\ p_2^{NE} = \dfrac{\bar{t}}{3}\big(v + 2(q_2 - q_1)\big) \end{cases}, \tag{3}$$

given that

$$v < 2\mathit{\Delta} q = 2(q_2 - q_1). \tag{4}$$

Proof. In order to find the second-stage price equilibrium we first construct the reaction functions (Tirole, 1988) of the firms.

Using (1) and (2) the low-quality firm reaction function can be derived as follows:

$$p_1(p_2) = \frac{1}{2}\left(1 + \frac{v}{v + 2\mathit{\Delta} q}\right) p_2. \tag{5}$$

The high-quality firm reaction function can be found at the same manner

$$p_2(p_1) = \frac{1}{2}\left[\left(1 + \frac{v}{v - 2\mathit{\Delta} q}\right) p_1 + \mathit{\Delta} q \cdot \bar{t}\left(1 - \frac{v}{v - 2\mathit{\Delta} q}\right)\right]. \tag{6}$$

Solving (5) and (6) as a system we get a unique solution (3). Note, that $0 < \dfrac{v\bar{t}}{3} < p_1^{NE} < p_2^{NE}$, and the vertical differentiation condition ($0 < \hat{t} < \bar{t}$) is satisfied.

When the Nash equilibrium prices p_1^{NE} and p_2^{NE} are selected the corresponding profit functions are equal to

$$\begin{cases} \mathit{\Pi}_1(q_1, q_2) = \dfrac{1}{18}\bar{t}\big(v + 2\mathit{\Delta} q\big) - FC(q_1) \\ \mathit{\Pi}_2(q_1, q_2) = \dfrac{2}{9}\bar{t}\big(2\mathit{\Delta} q - v\big) - FC(q_2) \end{cases}. \tag{7}$$

The condition (4) has to be taken into account to ensure that $\varPi_2 > 0$ (at least in the case when $FC(q) = 0$). □

4. Fixed costs and quality competition

In accordance to the subgame perfect equilibrium concept now we regard the first stage of the game — the stage of quality competition. To take into account fixed costs (the costs of quality improvement to ensure certain level of claims handling procedure) $FC(q)$ we'd like to consider 3 different types of $FC(q)$:

- zero fixed costs $FC(q) = 0$;
- linear fixed costs
$$FC(q) = \alpha q,\ \ \alpha > 0; \tag{8}$$
- quadratic fixed costs
$$FC(q) = \beta\frac{q^2}{2},\ \ \beta > 0. \tag{9}$$

4.1. Zero fixed costs

Obviously, in the case $FC(q) = 0$ the expected profit functions (7) are increasing in $\varDelta q = q_2 - q_1$:

$$\max_{q_1 \in [\underline{q}, \overline{q}]} \varPi_1(q_1, q_2) = \varPi_1(\underline{q}, q_2),$$

$$\max_{q_2 \in [\underline{q}, \overline{q}]} \varPi_2(q_1, q_2) = \varPi_2(q_1, \overline{q}).$$

Hence, in the case of zero fixed costs, the optimal behavior (in the sense of SPE) of the competitive firms implies the maximal level of product differentiation:

$$q_1^{NE} = \underline{q},\ q_2^{NE} = \overline{q}. \tag{10}$$

The corresponding optimal prices and the profits are equal to

$$\begin{cases} p_1^{NE} = \dfrac{\bar{t}}{3}\big(v + \overline{q} - \underline{q}\big) \\ p_2^{NE} = \dfrac{\bar{t}}{3}\big(v + 2(\overline{q} - \underline{q})\big) \end{cases}, \tag{11}$$

$$\begin{cases} \varPi_1^{NE} = \dfrac{\bar{t}}{18}\big(v + 2(\overline{q} - \underline{q})\big) \\ \varPi_2^{NE} = \dfrac{2\bar{t}}{9}\big(2(\overline{q} - \underline{q}) - v\big) \end{cases} \tag{12}$$

as far as the average variable cost parameter v is not too high, i.e. the condition (4) is satisfied: $v < 2(\overline{q} - \underline{q})$.

It's interesting to compare this subgame perfect equilibrium (10)–(12) with known optimal solution of the "seminal vertical differentiation model" (Tirole, 1988) and the vertical differentiation model in the insurance market, offered in (Okura, 2010). These models suppose zero costs of quality improvement, but differs slightly in the expressions of firms profit functions.

All three models imply the maximal quality differentiation. The optimal price differential in our model and seminal vertical differentiation model (Tirole, 1988) is the same: $p_2 - p_1 = \frac{1}{3}\bar{t}(\overline{q} - \underline{q})$, but the equilibrium prices in our model are lower than the equilibrium prices in (Tirole, 1988). Note, that the optimal price differential in (Okura, 2010) is not linear in $\bar{t}$ and $(\overline{q} - \underline{q})$.

The equilibrium expected profit differential in our model is lower than $\Delta\Pi = \Pi_2 - \Pi_1$ in the model of (Tirole, 1988). However in both models the high-quality firm gets larger profit than low-quality one (given that the average variable cost parameter v is small enough $v < \frac{6}{5}(\overline{q} - \underline{q})$ in our model). Note that the equilibrium expected profit differential in (Okura, 2010) nonlinearly depends on the parameters v, $\bar{t}$ and $\overline{q} - \underline{q}$.

4.2. Linear fixed costs

The expected profit functions (7) in the case of linear fixed costs (8) are as follows:

$$\begin{cases} \Pi_1(q_1, q_2) = \dfrac{1}{18}\bar{t}\left(v + 2q_2 - \left(2 + \dfrac{18\alpha}{\bar{t}}\right) q_1\right) \\ \Pi_2(q_1, q_2) = \dfrac{2}{9}\bar{t}\left(\left(2 - \dfrac{9\alpha}{2\bar{t}}\right) q_2 - 2q_1 - v\right) \end{cases}. \tag{13}$$

Again, the optimal quality for low-quality firm is $\underline{q}$, and the optimal quality for high-quality firm is $\overline{q}$ as far as coefficient α is small enough (to ensure the Π_2 is increasing in q_2, and $\Pi_2 > 0$):

$$\begin{cases} \alpha < \dfrac{4\bar{t}}{9} \\ \alpha < \dfrac{2\bar{t}\left(2(\overline{q} - \underline{q}) - v\right)}{9\overline{q}} \end{cases}. \tag{14}$$

4.3. Quadratic fixed costs

In this case $FC(q) = \dfrac{1}{2}\beta q^2$, and

$$\frac{\partial \Pi_1(q_1, q_2)}{\partial q_1} = -\frac{\bar{t}}{9} - \beta q_1 < 0,$$

hence, the low-quality firm should select $q_1^{NE} = \underline{q}$.

$$\frac{\partial \Pi_2(q_1, q_2)}{\partial q_2} = \frac{4\bar{t}}{9} - \beta q_2,$$

hence, the optimal level of q_2 depends on $\overline{q}$, $\bar{t}$ and β relations:

$$\begin{cases} q_2^{NE} = \dfrac{4\bar{t}}{9\beta}, \text{ if } \overline{q} > \dfrac{4\bar{t}}{9\beta} \\ q_2^{NE} = \overline{q}, \text{ if } \overline{q} \leq \dfrac{4\bar{t}}{9\beta} \end{cases}. \tag{15}$$

Note, that SPE in the case of quadratic fixed costs does not imply the maximal level of quality differentiation as far as the upper quality bound $\overline{q}$ is high enough (or the coefficient β is high enough).

5. Stakelberg (price) equilibrium

To find firms "optimal behavior" at the stage of price competition one can use different equilibria concepts or optimality principles (see, for instance (Hotelling, 1929; Petrosjan and Kuzyutin, 2008; Kuzyutin, 2012; Kuzyutin et al., 2014a and b).

In this section let us suppose the leader-follower behavior (Tirole, 1988; Petrosjan and Kuzyutin, 2008) at the stage of price competition and derive the Stakelberg price equilibrium. Namely, let the firm 1 be a leader, and a high quality firm 2 be a follower, i.e. firm 2 follows her reaction function (6) given the price p_1 chosen by the low-quality firm.

Taking this behavior into account the firm 1 tries to maximize the expected profit function

$$\mathit{\Pi}_1\big(p_1, p_2(p_1)\big) = \frac{(p_1 + \bar{t}\cdot \mathit{\Delta} q)\big(p_1(v - 4\mathit{\Delta} q) + v\bar{t}\cdot \mathit{\Delta} q\big)}{2\bar{t}(v - 2\mathit{\Delta} q)^2}. \tag{16}$$

This function has a unique maximum when

$$\frac{\partial \mathit{\Pi}_1}{\partial p_1} = \frac{p_1(v - 4\mathit{\Delta} q) + 2\bar{t}(q_2 - q_1)^2}{\bar{t}\big(v - 2(q_2 - q_1)\big)^2} = 0.$$

Thus the following statement holds.

Proposition 2. *In the QP-model (compulsory insurance case) there exists second-stage Stakelberg price equilibrium (firm 1 is a leader and firm 2 is a follower)*

$$\begin{cases} p_1^{SE} = \dfrac{2\bar{t}(q_2 - q_1)^2}{-\big(v - 4(q_2 - q_1)\big)} \\ p_2^{SE} = \dfrac{3}{2} p_1 \end{cases}, \tag{17}$$

given that $v < 4(q_2 - q_1)$.

To construct the SPE now we regard the stage of quality competition (in the simplest case of zero fixed costs $FC(q) = 0$).

Taking into account (17) the expected profit function of the low-quality firm is as follows

$$\mathit{\Pi}_1(q_1, q_2) = \frac{\bar{t}(q_2 - q_1)^2}{-2\big(v - 4(q_2 - q_1)\big)}.$$

$$\frac{\partial \mathit{\Pi}_1}{\partial q_1} = \frac{\bar{t}}{8}\left(\left(\frac{v}{v - 4(q_2 - q_1)}\right)^2 - 1\right) < 0,$$

given that

$$v < 2(q_2 - q_1) = 2\mathit{\Delta} q. \tag{18}$$

Hence, the optimal quality level of the firm 1 is $\underline{q}$ as far as condition (18) holds.

The expected profit function of the high-quality firm could be written in the following form:

$$\mathit{\Pi}_2(q_1, q_2) = -\frac{\bar{t}\big(v - 3(q_2 - q_1)\big)^2\big(v - 2(q_2 - q_1)\big)}{2\big(v - 4(q_2 - q_1)\big)^2}.$$

$$\frac{\partial \mathit{\Pi}_2}{\partial q_2} = -\frac{\bar{t}(5v - 12\mathit{\Delta} q)(v - 3\mathit{\Delta} q)(q_2 - q_1)}{(v - 4\mathit{\Delta} q)^3} > 0,$$

given that $v < \dfrac{12}{5}(q_2 - q_1)$.
Hence, the optimal quality level of the firm 2 is $\overline{q}$ as far as condition (18) holds.

Now let us compare the constructed Stakelberg equilibrium ($q_1^{SE} = \underline{q}$, $q_2^{SE} = \overline{q}$, optimal prices are determined in (17)) with the Nash Equilibrium case (10), (11).

Note that the optimal price differential $\Delta p^{SE} = p_2^{SE} - p_1^{SE}$ in the case of Stakelberg equilibrium is less than the optimal NE price differential $\Delta p^{NE} = p_2^{NE} - p_1^{NE}$ as far as $v < \overline{q} - \underline{q}$.

If we consider the following parameter's vector:

$$v = \frac{1}{2}; \quad \bar{t} = 1; \quad \underline{q} = 1; \quad \overline{q} = 3. \tag{19}$$

we can see that expected profits of both firms in the case of Stakelberg equilibrium are greater than in the case of Nash equilibrium and profit differential as well:

$$\begin{aligned} \Delta\Pi^{SE} &= \Pi_2^{SE} - \Pi_1^{SE} \approx 0,941 - 0,267 = 0,674, \\ \Delta\Pi^{NE} &= \Pi_2^{NE} - \Pi_1^{NE} \approx 0,778 - 0,25 = 0,528. \end{aligned}$$

6. The optional insurance case: monopoly pricing

From now we are going to change the assumption that the insurance service offered by the firms is compulsory for the consumers. Namely, we'd like to consider so called "optional insurance case" when each consumer can purchase one insurance service or reject the insurance services at all.

Let us start from the simplest case, when only one insurance firm offers the same insurance contract (q, p) to all the consumers. Each consumer's strategy in the considered optimal insurance case (monopoly settings without product differentiation) is maximizing his utility function of the following form:

$$u_t = \max\{0, tq - p\}.$$

The firm expected profit function is as follows

$$\Pi(q,p) = \frac{1}{\bar{t}}\left[\left(\bar{t} - \frac{p}{q}\right)p - \int_{\frac{p}{q}}^{\bar{t}} v \cdot t dt\right] = \frac{1}{\bar{t}}p^2\frac{v - 2q}{2q^2} + p - \frac{v}{2}\bar{t}. \tag{20}$$

Again, one can use the backwards induction procedure to derive optimal contract (q, p). Given q, satisfying the condition $v < 2q$ (which is similar to (18)) the expected profit function (20) has unique maximum in

$$p^{mon} = \frac{\bar{t}q^2}{2q - v}. \tag{21}$$

In the case of zero fixed costs the expected profit function $\Pi(q)$ is as follows

$$\Pi(q) = \frac{\bar{t}}{2}\left(\frac{q^2}{2q - v} - v\right). \tag{22}$$

The derivative

$$\frac{\partial \Pi}{\partial q} = \frac{\bar{t}q(q - v)}{(2q - v)^2}$$

is strictly positive as far as $v < q$. Hence the optimal quality for the firm is $\overline{q}$ if the condition

$$v < \overline{q} \tag{23}$$

is satisfied. Note that the same condition ensures the positive firm market share:

$$\overline{t} - \frac{p^{mon}}{\overline{q}} > 0.$$

Thus, the optimal monopoly behavior (in the case of optional insurance without product differentiation) is to offer the maximal quality $\overline{q}$, charging the price (21) as far as the condition (23) is satisfied.

Now let us consider the monopoly pricing when the firm offers two different insurance contracts (q_1, p_1) and (q_2, p_2), $\underline{q} \le q_1 < q_2 \le \overline{q}$, to the consumers. We'll call this scheme the optional insurance case (monopoly settings with product differentiation). The consumer t utility function is as follows

$$u_t = \max\{0, tq_1 - p_1, tq_2 - p_2\}.$$

The firm expected profit function can be written in the following form

$$\begin{aligned} \Pi(q_i, p_i) &= \frac{1}{\overline{t}} \left[\left(\frac{p_2 - p_1}{q_2 - q_1} - \frac{p_1}{q_1} \right) p_1 + \left(\overline{t} - \frac{p_2 - p_1}{q_2 - q_1} \right) p_2 - \int\limits_{\frac{p_1}{q_1}}^{\overline{t}} v \cdot t dt \right] = \\ &= \frac{1}{\overline{t}} \left[\frac{p_1^2 (v - 2q_1)}{2q_1^2} - \frac{(p_1 - p_2)^2}{q_2 - q_1} + \overline{t} p_2 - \frac{1}{2} v \overline{t}^2 \right]. \end{aligned} \tag{24}$$

To maximize expected profit (24) using backwards induction procedure we start from the second stage (the stage of optimal price's vector finding). Given q_2 and q_1, satisfying the inequality

$$v < 2q_1 \tag{25}$$

we can solve first order conditions:

$$\begin{cases} \dfrac{\partial \Pi(p_1, p_2)}{\partial p_1} = \dfrac{1}{\overline{t}} \left(\dfrac{p_1 (v - 2q_1)}{q_1^2} + \dfrac{2(p_2 - p_1)}{q_2 - q_1} \right) = 0 \\ \dfrac{\partial \Pi(p_1, p_2)}{\partial p_2} = 1 - \dfrac{2(p_2 - p_1)}{\overline{t}(q_2 - q_1)} = 0 \end{cases} . \tag{26}$$

The system (26) has a unique solution

$$\begin{cases} p_1 = -\dfrac{\overline{t} q_1^2}{v - 2q_1} \\ p_2 = \dfrac{\overline{t}}{2} \left(q_2 - \dfrac{v q_1}{v - 2q_1} \right) \end{cases} \tag{27}$$

which is the point of maximum of the expected profit function $\Pi(p_1, p_2)$.

Now let us regard the first stage (the stage of optimal qualities search). Again, we restrict ourselves to the case of zero fixed costs $FC(q) = 0$.

The expected profit function $\Pi(q_1, q_2)$ can be written in the following form

$$\Pi(q_1, q_2) = \frac{\bar{t}}{4}\left[q_2 - \frac{2v\left(v - \frac{3}{2}q_1\right)}{v - 2q_1}\right]. \tag{28}$$

Obviously, the derivative $\dfrac{\partial \Pi}{\partial q_1} = -\dfrac{v^2 \bar{t}}{4(v - 2q_1)^2}$ is strictly negative as far as the inequality (25) is satisfied, and $\dfrac{\partial \Pi}{\partial q_2} = \dfrac{\bar{t}}{4} > 0$. Hence, the optimal quality level q_1 of the low-quality firm is $\underline{q}$, the optimal level of q_2 is $\overline{q}$, and the optimal monopoly pricing (with product differentiation) implies the maximal level of vertical differentiation.

The resulting expressions for optimal contracts and profit are as follows:

$$\begin{cases} q_1^{mon} = \underline{q}, \; p_1^{mon} = -\dfrac{\bar{t}\underline{q}^2}{v - 2\underline{q}} \\ q_2^{mon} = \overline{q}, \; p_2^{mon} = \dfrac{\bar{t}}{2}\left(\dfrac{v(\overline{q} - \underline{q}) - 2\overline{q}\underline{q}}{v - 2\underline{q}}\right) \end{cases} \tag{29}$$

$$\Pi^{mon} = \frac{\bar{t}}{4}\left[\overline{q} - \frac{2v\left(v - \frac{3}{2}\underline{q}\right)}{v - 2\underline{q}}\right] \tag{30}$$

Let us note that $p_1^{mon} < p_2^{mon}$ as far as inequality $v < 2\underline{q}$ (or (25)) is satisfied.

When firm uses price differentiation it always gets higher profit than without differentiation:

$$\Pi^{mon}(\underline{q}, \overline{q}) - \Pi^{mon}(\overline{q}) = \frac{v^2 \cdot \bar{t}(\overline{q} - \underline{q})}{4(v - 2\overline{q})(v - 2\underline{q})} > 0$$

7. The optional insurance case in the duopoly settings

Let us suppose again that two insurance firms offer their insurance services (q_1, p_1) and (q_2, p_2) respectively, $0 < \underline{q} \le q_1 < q_2 \le \overline{q}$, and each consumer can purchase one product from a more desirable firm or reject the insurance services at all. We'll denote this product differentiation model by the QP-model (optional insurance case, duopoly settings).

To find subgame perfect equilibrium in this model we first have to derive the price Nash equilibrium at the second stage (the stage of price competition) given q_1 and q_2.

The expected profit functions are as follows:

$$\begin{aligned} \Pi_1(p_1, p_2) &= \frac{1}{\bar{t}}\left[\left(\hat{t} - \frac{p_1}{q_1}\right)p_1 - \int_{\frac{p_1}{q_1}}^{\hat{t}} vtdt\right]. \\ \Pi_2(p_1, p_2) &= \frac{1}{\bar{t}}\left[\left(\bar{t} - \hat{t}\right)p_2 - \int_{\hat{t}}^{\bar{t}} vtdt\right] \end{aligned}$$

Using first order conditions one can derive the reaction functions of the competitive firms:

$$\begin{cases} p_1(p_2) = -\dfrac{q_1^2(q_2 - q_1 + v)}{q_2\big(v(q_2 - 2q_1) - 2q_1(q_2 - q_1)\big)} \cdot p_2 \\ \\ p_2(p_1) = \dfrac{p_1(v - \Delta q) - \bar{t}(q_2 - q_1)^2}{v - 2\Delta q} \end{cases} \tag{31}$$

To use the reaction functions (31) we assume that inequality (4) and inequality $q_2 < 2q_1$ are satisfied.

Solving (31) we get a unique solution

$$\begin{cases} p_1^{NE} = \dfrac{\bar{t}q_1^2(v + q_2 - q_1)}{v^2 - q_1^2 - 2q_2(v - 2q_1)} \\ p_2^{NE} = \dfrac{\bar{t}q_2\Big(v(q_2 - 2q_1) - 2q_1(q_2 - q_1)\Big)}{-v^2 + q_1^2 + 2q_2(v - 2q_1)} \end{cases} \tag{32}$$

which is a price Nash equilibrium at the second stage (for some region of parameter values).

Taking (32) into account the expected profit functions $\Pi_i(q_1, q_2)$ in the case of zero fixed costs $FC(q) = 0$ at the stage of quality competition can be written in the following form:

$$\Pi_1(q_1, q_2) = \frac{\bar{t}(v - q_1)^2 q_2\big(2q_1(q_2 - q_1) - v(q_2 - 2q_1)\big)}{2\big(-v^2 + q_1^2 + 2q_2(v - 2q_1)\big)^2} \tag{33}$$

$$\Pi_2(q_1, q_2) = -\frac{\bar{t}(v - 2(q_2 - q_1))\big[q_1(v - 2q_2) + v(q_2 - v)\big]^2}{2\big(-v^2 + q_1^2 + 2q_2(v - 2q_1)\big)^2} \tag{34}$$

To derive the analytical expressions for quality equilibrium is a quite complicated problem. Let us find out the equilibrium qualities numerically for some values of parameters.

Namely, let us consider the parameter's vector (19). One can check that the quality pare

$$q_1^{NE} \approx 2,1538; \quad q_2^{NE} = \overline{q} = 3$$

forms Nash equilibrium, i.e.

$$\begin{aligned} \Pi_1(q_1^{NE}, q_2^{NE}) &= \max_{\underline{q} \le q_1 \le \overline{q}} \Pi_1(q_1, q_2^{NE}), \\ \Pi_2(q_1^{NE}, q_2^{NE}) &= \max_{\underline{q} \le q_2 \le \overline{q}} \Pi_2(q_1^{NE}, q_2). \end{aligned}$$

The corresponding equilibrium prices due to (32) are as follows

$$p_1^{NE} = 0,3383; \quad p_2^{NE} = 0,6987.$$

Thus, we managed to construct the subgame perfect equilibrium in the QP-model (optional insurance case, duopoly settings).

Note, that in the case of optional insurance (instead of compulsory insurance case) the subgame perfect equilibrium does not necessarily imply the maximal level of vertical product differentiation (even in the case of zero fixed costs).

It's interesting to compare the results obtained in duopoly settings with the optimal monopoly pricing (29), (30) for the case of two different insurance contracts.

Given parameters values (19) one can note that Nash price equilibrium in duopoly settings has at least three profitable for the consumers implications (comparing to the case of optimal monopoly pricing (29) with product differentiation):

- some consumers which reject any insurance service in monopoly settings now will purchase the lower-quality insurance product;
- the optimal low quality q_1^{NE} is higher than $q_1^{mon} = \underline{q}$;
- both prices p_1^{NE} and p_2^{NE} are much lower than optimal monopoly prices p_1^{mon} and p_2^{mon} correspondingly.

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