Ch 6. Product differentiation models

October 3, 2015

§6.4. Linear city model (Hotelling)

Assume that M consumers are uniformly distributed along line segment of length 1 (the "linear city") and competing firms are located at opposite sides of this segment.

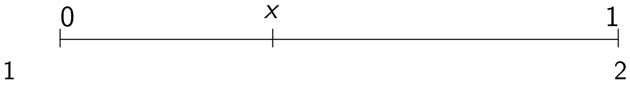

Fig. 6.4.1. Linear city model

Firms sell homogeneous good, marginal costs $c > 0$ are equal, asume zero fixed costs. All the consumers have the same willingness to pay (or rezervation price) $v > c$, each consumer will purshase no more than 1 item of the good.

There are different transport costs fo different consumers. For the consumer, located at x , trasport costs (if he buys from firm 1) are equal tx, trasport costs (if he buys from firm 2) – $t(1-x)$.
Namely, the travel costs generate the product differentiation (for every consumer $x \neq 1/2$).
The firm j can charge the price $p_j \in [c, v]$. Then each consumer $x \in [0, 1]$ has the following options:

- to purchase good from firm 1;
- to purchase good from firm 2;
- do not purchase.

The consumer $x \in [0, 1]$ surplus function:

$$U_x = max\{v - (p_1 + tx); v - (p_2 + t(1-x)); 0\}. \qquad (6.4.1)$$

Fig. 6.4.2 illustrates the consumers total costs functions $TC_1(x)$ $TC_2(x)$ and the purchase decisions of consumers located at various points for a given pair of prices:

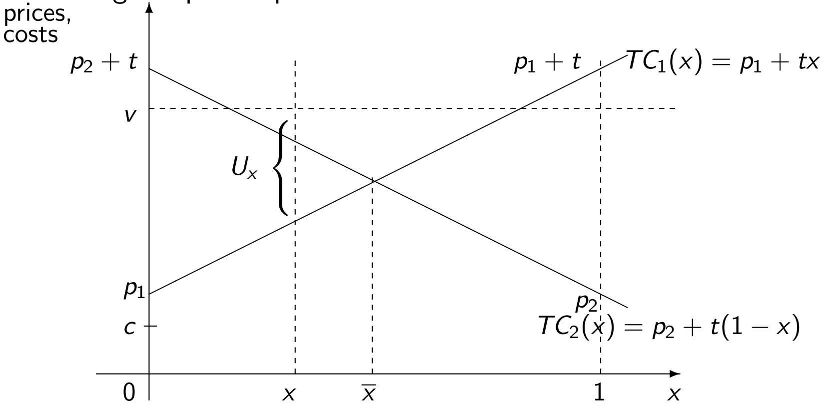

Fig. 6.4.2.

One can check that if the condition

$$v > c + 3t \tag{6.4.2}$$

holds the rational firms behavior implies that all the consumers will purchase the good from one of the firms. Let us find symmetric NE in this case:
"Indifferent" consumer:

$$\overline{x} = \frac{t + p_2 - p_1}{2t} \tag{6.4.3}$$

If $\overline{x} \in (0, 1)$, i.e. $|p_2 - p_1| < t$, The firms' demand functions have the following form:

$$\begin{cases} q_1(p_1, p_2) = M\overline{x} = (t + p_2 - p_1)\frac{M}{2t}; \\ q_2(p_1, p_2) = M(1 - \overline{x}) = (t + p_1 - p_2)\frac{M}{2t}. \end{cases} \tag{6.4.4}$$

The 1-st firm reaction function:

$$p_1 = R_1(p_2) = \frac{p_2 + t + c}{2}, \; p_2 \in (c, c + 3t). \tag{6.4.5}$$

The 2-nd firm reaction function:

$$p_2 = R_2(p_1) = \frac{p_1 + t + c}{2}, \; p_1 \in (c, c + 3t). \tag{6.4.6}$$

Hence, we get symmetric NE:

$$p_1^* = p_2^* = c + t. \tag{6.4.7}$$

Table 6.4.1

The NE structure in linear city model
depends on parameters (v, c, t) interrelation

N	parameters v, c and t	equilibrium prices	firms profits in symmetric *NE*
1	$v > c + \frac{3}{2}t$	$p_1^* = p_2^* = c + t$	$\pi_1^* = \pi_2^* = t \cdot \frac{M}{2}$
2	$v \in (c+t, c+\frac{3}{2}t)$	$p_1^* = p_2^* = v - \frac{t}{2}$	$\pi_1^* = \pi_2^* = (v - \frac{t}{2} - c) \cdot \frac{M}{2}$
3	$v \in (c, c + t)$	$p_1^* = p_2^* = \frac{v+c}{2}$	$\pi_1^* = \pi_2^* = \frac{(v-c)^2}{2t} \cdot \frac{M}{2}$

§6.5. Vertical differentiation model (monopoly settings)

The firm is a local monopoly, it can produce and sell good of any quality $q > 0$, the production of one unit of quality q costs $C(q)$. Let

$$C(q) = q^2, \; q > 0. \tag{6.5.1}$$

Each consumer plans to bye at most one unit of a good.
The parameter $t > 0$ indexes the consumer taste for quality (tq – maximal price the consumer is ready to pay for the good of quality q). Let there are only two possible values for t: t_1 and t_2, $0 < t_1 < t_2$.
Let us call the consumers of type t_2 "*sophisticated consumers*" (they are ready to pay more for quality increasing), and the consumers of type t_1 – "*coarse consumers*".

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Let us call the consumers of type t_2 "*sophisticated consumers*" (they are ready to pay more for quality increasing), and the consumers of type t_1 – "*coarse consumers*".

Assume that the consumers heterogeneity is not too high:

$$t_1 > \frac{t_2}{2}, \qquad (6.5.2)$$

The pair (q, p), where q – quality, and p – price of a good, is called *contract*. The firm's strategy is to choose two contracts (q_1, p_1) (q_2, p_2), one for sophisticated consumers and one for coarse, $q_1 < q_2$.

Then each consumer has the following options:

- to purchase good of high quality q_2 at price p_2;
- to purchase good of low quality q_1 at price p_1;
- do not purchase.

The consumer of type t_i, $i = 1, 2$, surplus function:

$$CS_i((q_1, p_1); (q_2, p_2)) = max\{t_i q_2 - p_2, t_i q_1 - p_1, 0\}. \qquad (6.5.3)$$

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First we will find the monopolist "optimal contracts" in the case of "perfect information and perfect discrimination". Assumptions:

- the firm can observe the type t_i of the consumer,
- the firm can offer only one contract to the consumer of given type.

The monopolist profit maximization problem allows the following decomposition: The first problem – to design the contract for coarse consumers:

$$\begin{cases} \pi_1(p_1, q_1) = p_1 - C(q_1) \longrightarrow \max_{(q_1,p_1)}; \\ t_1 q_1 - p_1 \geq 0 \end{cases} \tag{6.5.4}$$

The second – for the sophisticated consumers:

$$\begin{cases} \pi_2(p_2, q_2) = p_2 - C(q_2) \longrightarrow \max_{(q_2,p_2)}; \\ t_2 q_2 - p_2 \geq 0. \end{cases} \tag{6.5.5}$$

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For optimal contract (q_1^*, p_1^*) the constraint (inequality) should be binding: $p_1 = t_1 q_1$.
Therefore we just have to solve the problem:

$$\pi_1(q_1) = t_1 q_1 - C(q_1) = t_1 q_1 - q_1^2 \longrightarrow \max_{q_1}.$$

The optimal ("efficient") contract for coarse consumer:

$$\begin{cases} q_1^* = \frac{t_1}{2}; \\ p_1^* = t_1 q_1^* = \frac{t_1^2}{2}, \end{cases} \tag{6.5.6}$$

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Note that:

- all the consumers will purchase, but each consumer will get zero surplus;
- $q_1^* < q_2^*$.

Now let us turn to the case of "imperfect or asymmetric information":

- The firm only knows that the proportion of coarse consumers is α (let $\alpha = \frac{1}{2}$ for the simplicity);
- each consumer may choose any contract the firms offers at the market.

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The expected profit maximization problem (from a deal with one consumer):

$$\pi((q_1, p_1); (q_2, p_2)) = \frac{1}{2}(p_1 - q_1^2) + \frac{1}{2}(p_2 - q_2^2) \longrightarrow \max_{q_1, p_1, q_2, p_2}, \tag{6.5.8}$$

$$\begin{cases} t_1 q_1 - p_1 \geq t_1 q_2 - p_2; & (6.5.9) \\ t_2 q_2 - p_2 \geq t_2 q_1 - p_1; & (6.5.10) \\ t_1 q_1 - p_1 \geq 0; & (6.5.11) \\ t_2 q_2 - p_2 \geq 0. & (6.5.12) \end{cases}$$

The constraints (6.5.9) and (6.5.10) are called *incentive compatibility constraints*. They state that that each consumer prefers the conrtact that was designed for him.
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Suppose that the solution (q_1, p_1, q_2, p_2) of the problem (6.5.8) – (6.5.12), i.e. pair of optimal contracts (q_1, p_1) and (q_2, p_2) exists.

Theorem 6.5.1. For the pair of optimal contracts (q_1, p_1) and (q_2, p_2) the following 5 properties hold:

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$$t_2 q_2 - p_2 = t_2 q_1 - p_1; \tag{6.5.14}$$

- $q_2 \geq q_1$;
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Using Theorem 6.5.1 one can rewrite the original 4-variable maximization problem as a one-variable (q_1) maximization problem.

$$p_2 = t_2 q_2 - t_2 q_1 + p_1 = t_2 \cdot \frac{t_2}{2} - t_2 q_1 + t_1 q_1 = (t_1 - t_2) q_1 + \frac{t_2^2}{2}.$$

Then

$$\pi(q_1) = \frac{1}{2}\left(t_1 q_1 - q_1^2 + (t_1 - t_2) q_1 + \frac{t_2^2}{2} - \left(\frac{t_2}{2}\right)^2\right).$$

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Hence, we get the following pair of optimal contracts:

$$\begin{cases} q_1 = t_1 - \frac{t_2}{2}; & (6.5.16) \\ p_1 = t_1\left(t_1 - \frac{t_2}{2}\right); & (6.5.17) \\ q_2 = q_2^* = \frac{t_2}{2}; & (6.5.18) \\ p_2 = (t_1 - t_2)\left(t_1 - \frac{t_2}{2}\right) + \frac{t_2^2}{2}. & (6.5.19) \end{cases}$$

Note that the coarse consumer again will get zero surplus. However in the case of asymmetric information the sophisticated consumer will get positive surplus (which is called "his iformational rent").

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§6.6. Vertical differentiation model (duopoly settings)

Consider 2–stage game of vertical product differentiation (quality–price model or *QP*-model)

At the 1–st stage (the stage of quality choosing) $n = 2$ firms simultaneously choose their quality levels: $q_i \in [\underline{q}, \overline{q}] \subset [0, +\infty)$, facing quality improovement costs $FC(q)$.

At the 2–nd stage (the stage of price competition) the firms obtain information about chosen quality levels $(q_1, q_2), q_1 < q_2$, and then simultaneously choose prices p_1 and p_2.

Each consumer plans to bye at most one unit of a good. Total amount of consumers is S. The parameter $t > 0$ indexes the consumer taste for quality (tq – maximal price the consumer is ready to pay for the good of quality q), $t \in [\underline{t}, \overline{t}] \subset [0, +\infty)$. We assume the parameter t is uniformly distributed on $[\underline{t}, \overline{t}]$.

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Each consumer has the following options:

- to purchase good of quality q_1 at price p_1 from the 1–st firm;
- to purchase good of quality q_2 at price p_2 from the 2–nd firm;
- do not purchase.

The consumer of type t surplus function:

$$CS_t((q_1, p_1); (q_2, p_2)) = max\{t_i q_2 - p_2, t_i q_1 - p_1, 0\}. \quad (6.6.1)$$

The firms decisions generate "self–segmentation" of the consumers. Then one can evaluate the sales D_1 and D_2 and revenues for both firms. The variable costs $VC_i(q_i, D_i)$ now can be taken into account. Both firms tries to maximize own profit.

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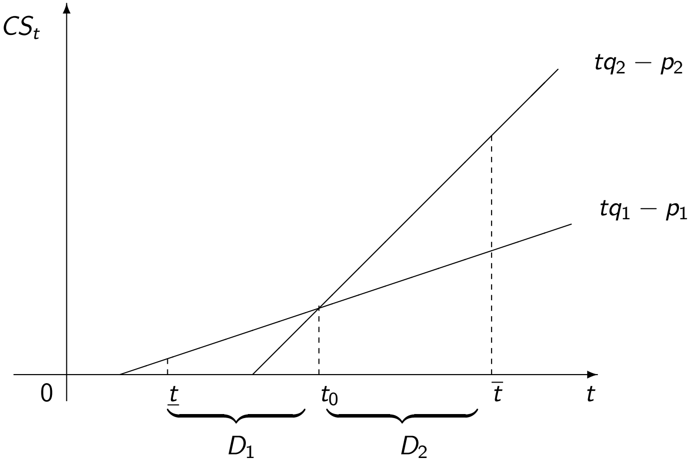

Fig. 6.6.1. Self–segmentation of the consumers

The indifference point $t_0 = \frac{p_2 - p_1}{q_2 - q_1}$ corresponds to the "indifferent consumer":

$$t_0 q_2 - p_2 = t_0 q_1 - p_1. \tag{6.6.2}$$

More sophisticated consumers $t \in (t_0, \overline{t}]$ will purchase from the 2–nd firm (given $(q_1, p_1; q_2, p_2)$)), more coarse consumers $t \in [\underline{t}, t_0)$ will purchase from the 1–st firm.

To simplify the analysis let us assume that:

$$\overline{t} - \underline{t} = 1, \tag{6.6.3}$$

$$S = 1, \tag{6.6.4}$$

$$FC(q) = 0, \tag{6.6.5}$$

$$VC_i(q_i, D_i) = VC_i(D_i) = c \cdot D_i, \tag{6.6.6}$$

where c – marginal cost (which does not depend on quality),

$$\overline{t} \geq 2\underline{t}, \tag{6.6.7}$$

$$c + \frac{\overline{t} - 2\underline{t}}{3}(q_2 - q_1) \leq \underline{t} q_1. \tag{6.6.8}$$

Constraint (6.6.7) implies that the consumers heterogeneity is high enough. Constraint (6.6.8) ensures that all the consumers will purchase in SPE.

To find SPE we start from the 2-nd stage (the stage of price competition). Assume that qualities q_1 and q_2 – have been already chosen at the 1–st stage.
Let $\Delta q = q_2 - q_1$. The firms sales will be:

$$D_1(q_1, p_1; q_2, p_2) = \frac{p_2 - p_1}{\Delta q} - \underline{t}; \tag{6.6.9}$$

$$D_2(q_1, p_1; q_2, p_2) = \overline{t} - \frac{p_2 - p_1}{\Delta q}. \tag{6.6.10}$$

The firms profits:

$$\pi_1(q_1, p_1; q_2, p_2) = (p_1 - c)(\frac{p_2 - p_1}{\Delta q} - \underline{t}); \tag{6.6.11}$$

$$\pi_2(q_1, p_1; q_2, p_2) = (p_2 - c)(\overline{t} - \frac{p_2 - p_1}{\Delta q}). \tag{6.6.12}$$

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Then we can derive the firms reaction functions (at the price competition stage):

$$p_1 = R_1(p_2) = \frac{1}{2}(p_2 + c - \underline{t} \cdot \Delta q); \qquad (6.6.13)$$

$$p_2 = R_2(p_1) = \frac{1}{2}(p_1 + c + \overline{t} \cdot \Delta q). \qquad (6.6.14)$$

To find NE we need to solve system (6.6.13), (6.6.14). The equilibrium prices:

$$\begin{cases} p_1^* = c + \frac{\overline{t}-2\underline{t}}{3} \cdot \Delta q; \\ p_2^* = c + \frac{2\overline{t}-\underline{t}}{3} \cdot \Delta q. \end{cases} \qquad (6.6.15)$$

Note that $p_1^* < p_2^*$, and all the consumers will purchase: $\frac{p_1^*}{q_1} \leq \underline{t}$ due to (6.6.8).

Then we can derive the firms reaction functions (at the price competition stage):

$$p_1 = R_1(p_2) = \frac{1}{2}(p_2 + c - \underline{t} \cdot \Delta q); \tag{6.6.13}$$

$$p_2 = R_2(p_1) = \frac{1}{2}(p_1 + c + \overline{t} \cdot \Delta q). \tag{6.6.14}$$

To find NE we need to solve system (6.6.13), (6.6.14). The equilibrium prices:

$$\begin{cases} p_1^* = c + \frac{\overline{t} - 2\underline{t}}{3} \cdot \Delta q; \\ p_2^* = c + \frac{2\overline{t} - \underline{t}}{3} \cdot \Delta q. \end{cases} \tag{6.6.15}$$

Note that $p_1^* < p_2^*$, and all the consumers will purchase: $\frac{p_1^*}{q_1} \leq \underline{t}$ due to (6.6.8).

Now we can use equilibrium prices (p_1^*, p_2^*) to get the fims profit functions at the 1–st stage:

$$\pi_1(q_1, q_2) = \frac{(\overline{t} - 2\underline{t})^2}{9} \cdot \Delta q = \frac{(\overline{t} - 2\underline{t})^2}{9} \cdot (q_2 - q_1), \quad (6.6.16)$$

$$\pi_2(q_1, q_2) = \frac{(2\overline{t} - \underline{t})^2}{9} \cdot \Delta q = \frac{(2\overline{t} - \underline{t})^2}{9} \cdot (q_2 - q_1). \quad (6.6.17)$$

Note that

$$\underline{q} \leq q_1 < q_2 \leq \overline{q}. \quad (6.6.18)$$

Each firm profit is proportional to the "quality differential" Δq, hence the 1–st firm will chose the lowest possible quality, and the 2–nd firm – the highest.

$$q_1^* = \underline{q},\ \ q_2^* = \overline{q}. \tag{6.6.19}$$

The resulting SPE in 2–stage game:

$$\begin{cases} q_1^* = \underline{q}, \\ q_2^* = \overline{q}, \\ p_1^*(\underline{q}, \overline{q}) = c + \frac{\overline{t} - 2\underline{t}}{3}(\overline{q} - \underline{q}), \\ p_2^*(\underline{q}, \overline{q}) = c + \frac{2\overline{t} - \underline{t}}{3}(\overline{q} - \underline{q}). \end{cases} \tag{6.6.20}$$

Therefore the firms "optimal behavior" (SPE) implies the maximal level of product differentiation in this simplest model.