Ch 6. Oligopoly

September 19, 2015

§6.1. Oligopoly: Bertrand competition

$TC_i(q_i) = c \cdot q_i$, $c > 0$ – marginal cost.
Linear demand function:

$$Q = D(p) = a - bp, p \in [0, \frac{a}{b}].$$

Demand function for the 1st firm:

$$q_1 = D_1(p_1, p_2) = \begin{cases} a - bp_1, & p_1 < p_2; \\ \frac{1}{2}(a - bp_1), & p_1 = p_2; \\ 0, & p_1 > p_2. \end{cases} \tag{6.1.1}$$

Payoff functions of competing firms:

$$\begin{cases} \pi_i(p_i, p_{3-i}) = (p_i - c) \cdot D_i(p_i, p_{3-i}) \longrightarrow \max_{p_i}; \\ p_i \geq 0. \end{cases} \tag{6.1.2}$$

Th 6.1.1.

There exists unique NE $p_1^* = p_2^* = c$ in Bertrand duopoly (oligopoly) symmetric model, and both firm's profits are equal zero (Bertrand paradox).

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Both firms simultaneously choose their outputs $q_1 \geq 0$ and $q_2 \geq 0$.
Total output: $q = q_1 + q_2$.
Inverse demand function:

$$p(q) = p(q_1 + q_2) = max\{a - bq, 0\}, \tag{6.2.1}$$

where a and b – positive parameters.
Marginal costs:

$$MC_1 = MC_2 = c,\ 0 \leq c < a, \tag{6.2.2}$$

assume zero fixed costs.

§6.2. Cournot duopoly (linear demand functions)

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Total output: $q = q_1 + q_2$.
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assume zero fixed costs.

The 1-st firm profit maximization problem:

$$\begin{cases} \pi_1(q_1, q_2) = q_1 \cdot p(q_1 + q_2) - cq_1 \longrightarrow \max\limits_{q_1}; \\ q_1 \geq 0. \end{cases} \tag{6.2.3}$$

Profit is positive iff:

$$a - b(q_1 + q_2) < c.$$

Then:

$$\begin{cases} \pi_1(q_1, q_2) = q_1 \cdot (a - c - b(q_1 + q_2)) \longrightarrow \max\limits_{q_1}; \\ 0 \leq q_1 \leq \frac{a-c}{b} - q_2. \end{cases} \tag{6.2.4}$$

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The problem (6.2.4) solution for given firm's 2 output $q_2 \in [0, \frac{a-c}{b})$ denote by $q_1 = R_1(q_2)$.

$$q_1 = R_1(q_2),\ q_2 \in [0, \frac{a-c}{b})$$

Reaction function (or *Best response function*) of the 1-st firm.

Given q_2 the 1-st firm reaction function $R_1(q_2)$ shows such 1-st firm output value q_1, which maximises her profit.

The problem (6.2.4) solution in explicit form:

$$q_1 = R_1(q_2) = \frac{a-c}{2b} - \frac{q_2}{2},\ 0 \le q_2 < \frac{a-c}{b}. \qquad (6.2.5)$$

If $q_2 \ge \frac{a-c}{b}$ let $R_1(q_2) = 0$.

The 2-nd firm reaction function $q_2 = R_2(q_1)$:

$$q_2 = R_2(q_1) = \frac{a-c}{2b} - \frac{q_1}{2},\ 0 \le q_1 < \frac{a-c}{b}. \qquad (6.2.6)$$

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The graphs of reaction functions - *Reaction curves*.

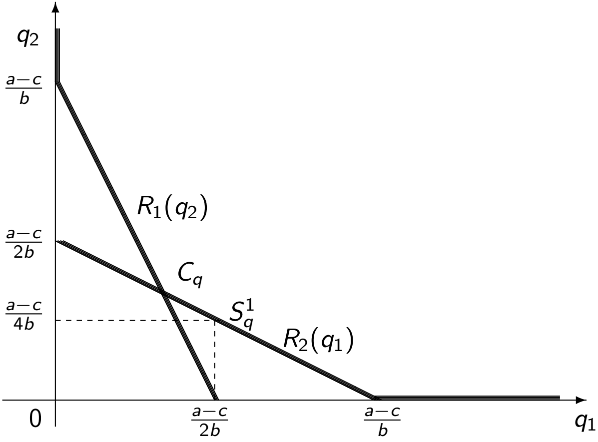

Fig. 6.2.1. Reaction curves, Cournot and Stakelberg equilibriums (in strategy space Oq_1q_2)

The solution (q_1^*, q_2^*) of the system (6.2.5) and (6.2.6) is called *Cournot equilibrium*.

$$q_1^* = q_2^* = \frac{a-c}{3b}. \tag{6.2.7}$$

Note that:

$$p^* = p(q_1^* + q_2^*) = \frac{1}{3}(a + 2c) \tag{6.2.8}$$

– market price,

$$\pi_1^c = \pi_2^c = \pi_i(q_1^*, q_2^*) = \frac{(a-c)^2}{9b} \tag{6.2.9}$$

– the firm profit.

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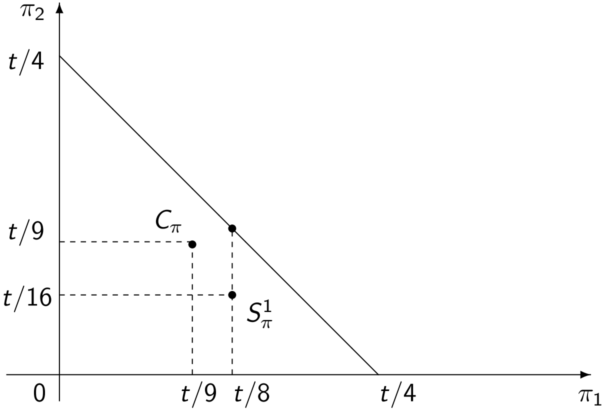
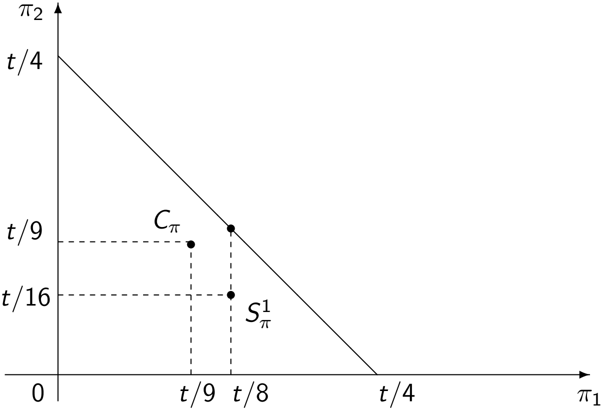


Fig. 6.2.2. Cournot and Stackelberg equilibriums (in payoff functions space $O\pi_1\pi_2$), $\frac{(a-c)^2}{b} = t$

Property 6.2.1. Cournot equilibrium (q_1^*, q_2^*) is NE.



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Stackelberg model (quantity leadership).
Consider a two-stage game in which one firm (leader) gets to move first. The other firm (follower) then observes the leader's output and chooses its own output using best response function.

Let firm 1 be the leader and firm 2 be the follower.

The leader's profit maximization problem:

$$\begin{cases} \pi_1(q_1, R_2(q_1)) \longrightarrow \max_{q_1}; \\ q_1 \geq 0. \end{cases} \tag{6.3.1}$$

The solution $\bar{q}_1$ of (6.3.1) – the leader's optimal output, $\bar{q}_2 = R_2(\bar{q}_1)$ – the follower's optimal output, $(\bar{q}_1, \bar{q}_2)$ – *Stackelberg equilibrium.*

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In the case of linear demand:

$$\bar{q}_1 = \frac{a-c}{2b}, \ \bar{q}_2 = R_2(\bar{q}_1) = \frac{a-c}{4b}. \tag{6.3.2}$$

$$q = \frac{3(a-c)}{4b},$$

$$\bar{p} = \frac{a+3c}{4} \tag{6.3.3}$$

– market price in Stackelberg model,

$$\pi_1^S = \frac{(a-c)^2}{8b}, \ \pi_2^S = \frac{(a-c)^2}{16b} \tag{6.3.4}$$

– the firms profits.

Other possible patterns of firm behavior (duopoly settings):

- collusion;
- both firms choose their output like a follower in Stackelberg model;
- both firms choose their output like a leader in Stackelberg model.

The collusion scheme:

$$q^m = q_1 + q_2 = \frac{a-c}{2b}, q_1 \geq 0, q_2 \geq 0,$$

optimal monopoly price $p_m = \frac{a+c}{2}$, maximal total profit

$$\pi^m = \pi_1 + \pi_2 = \frac{(a-c)^2}{4b}.$$

There are many strategy profiles (q_1, q_2), that satisfy $(q_1 + q_2 = q^m)$, but no profile satisfies NE.

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The followers scheme:

$$q_1 = q_2 = \frac{a-c}{4b}.$$

Total output $q = \frac{a-c}{2b}$, market prise coincides with monopoly price $p_m = \frac{a+c}{2}$, each firm profit:

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The leaders scheme:

$$q_1 = q_2 = \frac{a-c}{2b}.$$

Total output: $q = \frac{a-c}{b}$, market price: $p = c$, both firms get zero profit.