Ch 5. Monopoly behavior

September 11, 2015

§5.1. Monopoly pricing

The simple model of monopoly behavior:

$$\begin{cases} \pi(q) = p(q) \cdot q - TC(q) \longrightarrow max; \\ q \geq 0. \end{cases} \tag{5.1.1}$$

First order conditions:

$$MC(q^m) = MR(q^m) = p(q^m) + p'(q^m) \cdot q^m \tag{5.1.2}$$

$$p^m = p(q^m) > MC(q^m). \tag{5.1.3}$$

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Taking into account the relationship of MR and demand elasticity we get first order condition in the following form:

$$MC(q^m) = MR(q^m) = p^m \left(1 + \frac{1}{E(p^m)}\right). \quad (5.1.4)$$

The monopoly "works" at such point of demand curve where

$$E(p^m) = E(p(q^m)) < -1. \quad (5.1.5)$$

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§5.2. Price discrimination

Perfect price discrimination means that monopolist sells each item of the good at maximal price the consumer is ready to pay.

Examples

Real Estate service (Commission fee equals certain percent of the property price).
Notary service (Inheritance tax equals certain percent of the inherited property value).

Example 5.2.1. *Compare perfect competition, monopoly pricing and perfect price discrimination* (linear demand case, constant MC) Let

$$q(p) = 48 - 2p, p \in [0, 24]$$

– market demand;

$$TC(q) = 2q$$

– total cost function.

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"Perfect competition": $MC = 2$, efficient output level and market price: $q^c = 44, p^c = 2$ (fig. 5.2.1).

Fig. 5.2.2. Perfect price discrimination

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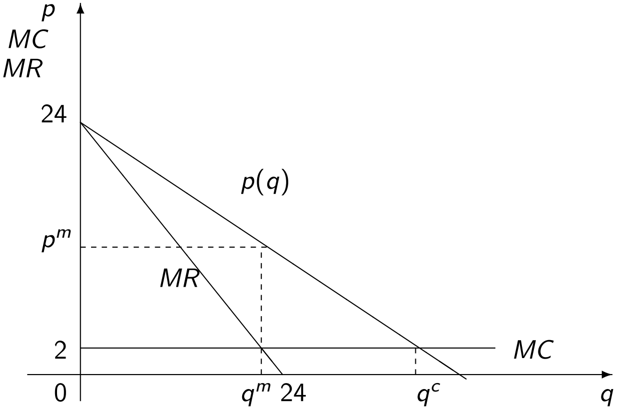

Fig. 5.2.2. Perfect price discrimination

Inverse demand function $p(q) = 24 - \frac{1}{2}q$, marginal revenue $MR(q) = 24 - q$.

Monopoly pricing:

$$q^m = 22,\ p^m = 13,\ \pi(q^m) = 42;$$
$$\pi(p^m) = 242;$$
$$CS(p^m) = 22 \cdot 11 \cdot \tfrac{1}{2} = 121.$$

"Deadweight Welfare Loss" ?

Perfect price discrimination:

Actual price will be different for different consumers (from 24 to 2), output increases up to $q^c = 44$, every consumer gets zero consumer's surplus , the monopolist profit rises to its maximal possible value:

$$\pi(p^m) = 44 \cdot 22 \cdot \frac{1}{2} = 484.$$

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Second-degree price discrimination
(nonlinear pricing or quantity discounts)

Third-degree price discrimination
occures when consumers are charged different prices (but each consumer faces a constant price for all units of output purchased).
Consider two separate markets (price elasticity differs !).
Let $p_1(q_1)$ and $p_2(q_2)$– inverse demand functions,
$TC(q_1 + q_2) = TC(q)$ – total cost function.
The monopolist profit maximization problem:

$$\begin{cases} \pi(q_1, q_2) = q_1 \cdot p_1(q_1) + q_2 \cdot p_2(q_2) - C(q_1 + q_2) \longrightarrow max, \\ q_1 \geq 0, q_2 \geq 0. \end{cases} \tag{5.2.1}$$

Notice that:

$$\frac{\partial TC}{\partial q_1} = \frac{\partial TC}{\partial q_2} = \frac{\partial TC}{\partial q} = MC(q) = MC(q_1 + q_2). \tag{5.2.2}$$

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$$\begin{cases} MR(\tilde{q}_1) = MC(\tilde{q}_1 + \tilde{q}_2); \\ MR(\tilde{q}_2) = MC(\tilde{q}_1 + \tilde{q}_2). \end{cases} \tag{5.2.3}$$

Note: $\tilde{p}_i = p_i(\tilde{q}_i)$
Then:

$$\tilde{p}_1 \left(1 + \frac{1}{E_1(\tilde{p}_1)}\right) = \tilde{p}_2 \left(1 + \frac{1}{E_2(\tilde{p}_2)}\right). \tag{5.2.4}$$

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§5.3. Example (third-degree price discrimination versus monopoly pricing)

The demand function at 1-st market:

$$q_1(p_1) = 96 - p_1, \; p_1 \leq 96,$$

at 2-nd market:

$$q_2(p_2) = 120 - 2p_2, \; p_2 \leq 60;$$

total cost function: $TC(q) = q^2$.
Consider third-degree price discrimination.

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$$\begin{cases} 96 - 2\tilde{q}_1 = 2(\tilde{q}_1 + \tilde{q}_2); \\ 60 - \tilde{q}_2 = 2(\tilde{q}_1 + \tilde{q}_2). \end{cases}$$

The unique solution (point of maximum):
$\tilde{q}_1 = 21, \tilde{q}_2 = 6.$
Corresponding optimal prices:
$\tilde{p}_1 = 75$ and $\tilde{p}_2 = 57$,
The monopolist profit: $\pi(\tilde{q}_1, \tilde{q}_2) = 1188.$

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Table 5.3.1

Third-degree price discrimination

	Demand functions	Inverse demand functions	Optim. prices	Optim. outputs .	$E_i(\tilde{p}_i)$
Market 1	$q_1(p_1) = 96 - p_1,$ $p_1 \leq 96$	$p_1(q_1) = 96 - q_1$	$\tilde{p}_1 = 75$	$\tilde{q}_1 = 21$	$-\frac{75}{21}$
Market 2	$q_2(p_2) = 120 - 2p_2,$ $p_2 \leq 60$	$p_2(q_2) = 60 - \frac{1}{2}q_2$	$\tilde{p}_2 = 57$	$\tilde{q}_2 = 6$	$-\frac{57}{3}$

The 2-nd market implies more elastic demand, and monopolist charges lower price at this market.
The total sales amount $\tilde{q}_1 + \tilde{q}_2 = 27$.

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Now consider the same price at each market (simple monopoly pricing).

The aggregated market demand function:

$$q(p) = \begin{cases} 216 - 3p, & p \in [0, 60]; \\ 96 - p, & p \in [60, 96]. \end{cases} \quad (5.3.1)$$

Optimal monopoly price: $p^m = 72$,
sales amounts are: $q_1^m = 24 \quad \tilde{q}_2 = 0$.

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	Demand function	Optim. monopoly price	Optim. sales amounts
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If the price discrimination does not take place, the 2-nd market consumers will not purchase the good (the price is too high). Thus, the price discrimination "revives" the 2-nd market in this example.

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