NASA TECHNICAL TRANSLATION NASA TT F-15,813

TASK OF PREDICTING THE ROTATIONAL MOTION OF AN UNSYMMETRICAL TWISTED ARTIFICIAL EARTH SATELLITE

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Translation of "Zadacha prognozirovaniya vrashchatel'nogo dvizheniya nesimmetrichnogo zakruchennogo ISZ," in: Mekhanika upravlyayemogo dvizheniya i problemy kosmicheskoy dinamiki, Ed. by V. S. Novoselov, Leningrad University Press, Leningrad, 1972, pp. 103-113.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546 AUGUST 1974

STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TT F-15813		

4. Title and Subtitle	5. Report Date
TASK OF PREDICTING THE ROTATIONAL MOTION OF AN UNSYMMETRICAL TWISTED ARTIFICIAL EARTH SATELLITE	August 1974
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
V. S. Novoselov, L. K. Babadzhanyants, L. I. Fedorova	10. Work Unit No.
9. Performing Organization Name and Address	11. Contract or Grant No.
Leo Kanner Associates Redwood City, California 94063	NASw-2481-407
	13. Type of Report and Period Covered
	Translation
12. Sponsoring Agency Name and Address	
National Aeronautics and Space Administration, Washington, D. C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes

Translation of "Zadacha prognozirovaniya vrashchatel'nogo dvizheniya nesimmetrichnogo zakruchennogo ISZ," in: Mekhanika upravlyayemogo dvizheniya i problemy kosmicheskoy dinamiki, Leningrad University Press, Leningrad, 1972, pp. 103-113.

16. Abstract

Equations using osculating elements in the general case of dynamically unsymmetrical bodies are discussed. The case in point is that of an artificial Earth satellite in a circular orbit of an altitude on the order of 700 km. Two problems must be solved: refinement of the parameters and initial values; numerical integration in a given time interval. The Runge-Haine-Kutta method is tested as it applies to derivative-unsolved systems.

17. Key Words (Selected by Author(s))	18. Distribution Statement
	Unclassified-Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	17	

NASA-HQ

TASK OF PREDICTING THE ROTATIONAL MOTION OF AN UNSYMMETRICAL TWISTED ARTIFICIAL EARTH SATELLITE

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Equations using osculating elements L, ρ, σ, θ, ϕ, ψ in the general case of dynamically unsymmetrical bodies are discussed: /103

$$\dot{L} = M_3, \tag{1}$$

$$\dot{\rho} = \frac{1}{L} M_1, \tag{2}$$

$$\dot{\sigma} = \frac{1}{L \sin \rho} M_2, \tag{3}$$

$$\dot{\vartheta} = L \sin \vartheta \sin \varphi \cos \varphi \left(\frac{1}{I_x} - \frac{1}{I_y} \right) + \frac{M_2 \cos \psi - M_1 \sin \psi}{L}, \tag{4}$$

$$\dot{\varphi} = L \cos \vartheta \left(\frac{1}{I_z} - \frac{\sin^2 \varphi}{I_x} - \frac{\cos^2 \varphi}{I_y} \right) + \frac{M_1 \cos \psi + M_2 \sin \psi}{L \sin \vartheta}, \tag{5}$$

$$\dot{\psi} = L \left(\frac{\sin^2 \varphi}{I_x} + \frac{\cos^2 \varphi}{I_y} \right) - \frac{M_1 \cos \psi + M_2 \sin \psi}{L \sin \vartheta} \cos \vartheta - \frac{M_2}{L} \operatorname{ctg} \rho. \tag{6}$$

The derivation of these equations was given by F. L. Chernous'ko (cf. [1]). The meaning of the letters appearing in (1)-(6) is the same as in V. V. Beletskiy's book (cf. [2]). We will not stop to discuss why these particular equations were selected; the advantage of these equations for the case of fast-twisted artificial Earth satellites is discussed in detail in Belitskiy's book.

Equations (1)-(6) reflect the probably (in some sense) actual rotation of the AES in proportion to the closeness to true moments M_1, M_2, M_3, where $\vec{M}$ is the perturbing moment of all external forces.

In order to predict AES rotation, we must be able to solve two equations. /104

1. To find a sufficiently close to actual vector-function for perturbing moments $\vec{M}(t)$ (time t can appear evidently or thanks to $L, \rho, \sigma, \theta, \phi, \psi, \dot{L}, \dot{\rho}, \dot{\sigma}, \dot{\theta}, \dot{\phi}, \dot{\psi}$).

2. To integrate the system (1)-(6) for a sufficiently large time interval and sufficiently accurately.

The discussion is being conducted for the case of an AES in circular orbits of an altitude on the order of 700 km.

2. Theoretical Formulas for Perturbing Moments

We find that

$$\vec{M} = \| \alpha \| \vec{m}, \tag{7}$$

where $\vec{M}=(M_1, M_2, M_3)$, $\vec{m}=(M_x, M_y, M_z)$, a $\| \alpha \| = \begin{pmatrix} \alpha_{11} & \dots & \alpha_{13} \\ \cdot & \cdot & \cdot \\ \alpha_{31} & \dots & \alpha_{33} \end{pmatrix}$

are a matrix of direction cosines.

The system (x, y, z) is rigidly connected with the satellite. Components a_{ij} are expressed by the Euler angles θ, ψ and φ with the aid of formulas (1.1.5) from Beletskiy's book (cf. [1], p. 21). As in study [3], let us write

$$M_x = \frac{1}{981} (M_x^{\text{мт}} + M_x^{\text{мм}} + M_x^{\text{в}}) + M_x^{\text{к}} + M_x^{\text{а}} + L_x, \tag{8}$$

$$M_y = \frac{1}{981} (M_y^{\text{мт}} + M_y^{\text{мм}} + M_y^{\text{в}}) + M_y^{\text{к}} + M_y^{\text{а}} + L_y, \tag{9}$$

$$M_z = \frac{1}{981}\left(M_z^{\text{жт}} + M_z^{\text{мж}} + M_z^{\text{в}}\right) + M_z^{\text{к}} + M_z^{\text{п}} + L_z. \quad (10)$$

Perturbing moments from permanent-magnet iron:

$$M_x^{\text{жт}} = D_y^{\text{т}} H_z - D_z^{\text{т}} H_y, \quad (11)$$

(12)

$$M_z^{\text{жт}} = D_x^{\text{т}} H_y - D_y^{\text{т}} H_x. \quad (13)$$

Perturbing moments from soft-magnet iron:

$$M_x^{\text{мж}} = k_1 H_y H_z, \qquad k_1 = k_{22} - k_{33}, \quad (14)$$

$$M_y^{\text{мж}} = k_2 H_x H_z, \qquad k_2 = k_{33} - k_{11}, \quad (15)$$

$$M_z^{\text{мж}} = k_3 H_x H_y, \qquad k_3 = k_{11} - k_{22}. \quad (16)$$

Moments from eddy currents:

$$M_x^{\text{в}} = -k_x \omega_x (H_y^2 + H_z^2) + k_y \omega_y H_x H_y + k_z \omega_z H_x H_z, \quad (17)$$

$$M_y^{\text{в}} = k_x \omega_x H_x H_y - k_y \omega_y (H_x^2 + H_z^2) + k_z \omega_z H_y H_z, \quad (18)$$

$$M_z^{\text{в}} = k_x \omega_x H_x H_z + k_y \omega_y H_y H_z - k_z \omega_z (H_x^2 + H_y^2). \quad (19)$$

Perturbing moments from uncompensated kinetic momentum: /105

$$M_x^{\text{к}} = \omega_z L_y - \omega_y L_z, \quad (20)$$

$$M_y^{\text{к}} = \omega_x L_z - \omega_z L_x, \quad (21)$$

$$M_z^{\text{к}} = \omega_y L_x - \omega_x L_y. \quad (22)$$

Gravitational perturbing moments:

$$L_x = 3\omega_0^2 (I_z - I_y) a_{13} a_{12}, \quad (23)$$

$$L_y = 3\omega_0^2 (I_x - I_z) a_{11} a_{13}, \quad (24)$$

$$L_z = 3\omega_0^2 (I_y - I_x) a_{12} a_{11}. \quad (25)$$

Aerodynamic perturbing moments:

$$M_x^a = \frac{1}{2} c\rho v_0 [-I_1 \omega_x + I_2 (a_{23}\omega_y - a_{22}\omega_z), \quad (26)$$

$$M_y^a = \frac{1}{2} \rho v_0^2 (a_0^6 + a_2^6 a_{21}^2) a_{23} + \frac{1}{2} c\rho v_0 (a_{23} I_4 \omega_x - I_3 \omega_y), \quad (27)$$

$$M_z^a = -\frac{1}{2} \rho v_0 (a_0^6 + a_2^6 a_{21}^2) a_{22} + \frac{1}{2} c\rho v_0 (-a_{22} I_4 \omega_x - I_3 \omega_z). \quad (28)$$

Projections of geomagnetic field intensity into connected axes are defined in the form

$$H_x = H_{x1} a_{11} + H_{y1} a_{21} + H_{z1} a_{31}, \quad (29)$$

$$H_y = H_{x1} a_{12} + H_{y1} a_{22} + H_{z1} a_{32}, \quad (30)$$

$$H_z = H_{x1} a_{13} + H_{y1} a_{23} + H_{z1} a_{33}. \quad (31)$$

For a circular satellite orbit of about 700 km altitude, we can assume in oersteds that

$$H_{x1} = -0{,}630 \left(\frac{R_3}{R_3 + h_c}\right)^3 \sin i \sin \omega_0 t, \quad (32)$$

$$H_{y1} = 0{,}315 \left(\frac{R_3}{R_3 + h_c}\right)^3 \sin i \cos \omega_0 t, \quad (33)$$

$$H_{z1} = 0{,}315 \left(\frac{R_3}{R_3 + h_c}\right)^3 \cos i. \quad (34)$$

Projections of angular velocity of satellite rotation will be written thus:

$$\omega_x = \dot{\rho}\alpha_{21} + \dot{\sigma}(\alpha_{31}\cos\rho - \alpha_{11}\sin\rho) + \dot{\vartheta}\cos\varphi + \dot{\psi}\alpha_{31}, \tag{35}$$

$$\omega_y = \dot{\rho}\alpha_{22} + \dot{\sigma}(\alpha_{32}\cos\rho - \alpha_{12}\sin\rho) - \dot{\vartheta}\sin\varphi + \dot{\psi}\alpha_{32}, \tag{36}$$

$$\omega_z = \dot{\rho}\alpha_{23} + \dot{\sigma}(\alpha_{33}\cos\rho - \alpha_{13}\sin\rho) + \dot{\varphi} + \dot{\psi}\alpha_{33}. \tag{37}$$

The derivation of formulas (7)-(37) is given in study [3]. The same notations are used.

Expressions for a_{ij} through L, ρ, σ, φ, ψ, θ, t are easily derived by using formulas (1.1.3) and (1.1.4) from Beletskiy's book ([1], pp. 20-22). We find that

$$a_{11} = \gamma_1 \cos\omega_0 (t - t_{\Omega}) + \alpha_1 \sin\omega_0 (t - t_{\Omega}). \tag{38}$$

$$a_{12} = \gamma_2 \cos\omega_0 (t - t_{\Omega}) + \alpha_2 \sin\omega_0 (t - t_{\Omega}), \tag{39}$$

$$a_{13} = \gamma_3 \cos\omega_0 (t - t_{\Omega}) + \alpha_3 \sin\omega_0 (t - t_{\Omega}). \tag{40}$$ /106

$$a_{21} = \alpha_1 \cos\omega_0 (t - t_{\Omega}) - \gamma_1 \sin\omega_0 (t - t_{\Omega}), \tag{41}$$

$$a_{22} = \alpha_2 \cos\omega_0 (t - t_{\Omega}) - \gamma_2 \sin\omega_0 (t - t_{\Omega}), \tag{42}$$

$$a_{23} = \alpha_3 \cos\omega_0 (t - t_{\Omega}) - \gamma_3 \sin\omega_0 (t - t_{\Omega}), \tag{43}$$

$$a_{31} = \beta_1, \tag{44}$$

$$a_{32} = \beta_2, \tag{45}$$

$$a_{33} = \beta_3, \tag{46}$$

$$\alpha_1 = m_1\alpha_{11} + m_2\alpha_{21} + m\alpha_{31}, \tag{47}$$

$$\alpha_2 = m_1\alpha_{12} + m_2\alpha_{22} + m\alpha_{32}, \tag{48}$$

$$\alpha_3 = m_1\alpha_{13} + m_2\alpha_{23} + m\alpha_{33}, \tag{49}$$

$$\beta_1 = n_1\alpha_{11} + n_2\alpha_{21} + n\alpha_{31}, \tag{50}$$

$$\beta_2 = n_1\alpha_{12} + n_2\alpha_{22} + n\alpha_{32}, \tag{51}$$

$$\beta_3 = n_1\alpha_{13} + n_2\alpha_{23} + n\alpha_{33}, \tag{52}$$

$$\gamma_1 = k_1\alpha_{11} + k_2\alpha_{21} + k\alpha_{31}, \tag{53}$$

$$\gamma_2 = k_1\alpha_{12} + k_2\alpha_{22} + k\alpha_{32}, \quad (54)$$

$$\gamma_3 = k_1\alpha_{13} + k_2\alpha_{23} + k\alpha_{33}, \quad (55)$$

$$m_1 = \cos\rho \sin\sigma, \quad (56)$$

$$m_2 = \cos\sigma, \quad (57)$$

$$m = \sin\rho \sin\sigma, \quad (58)$$

$$n_1 = -\sin\rho, \quad (59)$$

$$n_2 = 0, \quad (60)$$

$$n = \cos\rho, \quad (61)$$

$$k_1 = \cos\rho \cos\sigma, \quad (62)$$

$$k_2 = -\sin\sigma, \quad (63)$$

$$k = \sin\rho \cos\sigma. \quad (64)$$

As was already noted in section 1, the problem of predicting consists of I and II problems, whose solution requires the specification of the value of initial data L(0), ..., $\psi(0)$, and also the parameters k_1, k_2, k_3, k_x, k_y, k_z, D_x^T, D_y^T, D_z^T (or several of them) which are known in coarse approximation. Thereafter, the problem is solved by numerical integration of the aforementioned equations with fixed initial conditions and known right sides.

Therefore, we can finally assert that two problems must be solved.

A. Specification of the parameters and initial values.

B. Numerical integration in a given time interval.

But to solve problem A, we need a reiterated solution of problem B in which we also uncover the theoretical complexity. The fact of the matter is that in the case of a fast-twisted AES, in equations (1)-(6) the variables ϕ and ψ are rapidly varying functions of time t. Thus the step-by-step numerical integration /107
of (1)-(6) (due to the small spacing on the order of 0.2-2.5 s) in an extended interval requires a great deal of computer time.

This work will study two appraoches to the problem of predicting orbital motion of a twisted nonsymmetrical AES: prediction in a short time interval by precise (non-averaged) equations and prediction in a long time interval by averaging with respect to ϕ and ψ.

3. Mathematical algorithm of integrating equations of orbital motion of a nonsymmetrical satellite in a short time interval

1°. In the right sides of equations (1)-(6) appear the moments M_1, M_2, M_3. These moments are written in clear form with the aid of formulas (7)-(64), while formulas (35)-(37) contain derivatives of the unknown functions. All (known to us) numerical methods of solving systems of differential equations of the first order have been developed for the case of the right sides in which derivatives of unknown functions do not appear.

It goes without saying that we can easily apply differential methods, especially extrapolatory, to these systems employing an interative method of defining the derivatives at each step. We, however, will not employ these methods both because of the enormous complexity involved in compiling the tables and because the substance of the problem under investigation requires frequent variation of the spacing of integration during the program.

The Runge-Haine-Kutta method is free of these shortcomings, but its application to derivative-unsolved systems causes many problems. These problems consist of the fact that at each step we will have to four-fold apply the iteration method. Of course, in virtue of the linearity of M_1, M_2, $M_3$with respect to all derivatives, we can resolve system (1)-(6) with respect to the latter. This path, however, is also unsuitable for us for the folreasons.

Given that to calculate each right side of system (1)-(6) we must have p operations. In resovling this system relative to the derivatives, we will find a new system whose right sides contain determinants of the 6th order. To calculate each element of this determinant will require, on the average, p/6 operations; the total to calculate (precisely) the determinant on an average (6!) · p/6 = 120 p operations. We can naturally calculate the determinant roughly, but as we can easily see, this is virtually the same as using the iteration method mentioned above. Thus, since ω_x, ω_y and ω_z are functions of derivatives with respect to /108
which the equations are resolved, this problem becomes much more complicated.

2°. Below is stated a method which allows us to avoid the problems mentioned above.

a) The values L_0, $\dot{\rho}_0$, $\dot{\sigma}_0$, $\dot{\theta}_0$, $\dot{\phi}_0$, $\dot{\psi}_0$ at the initial point are derived from system (1)-(6) by the iteration method; for the initial approximations of these quantities we take quantities equal to the right sides, if in the latter instead of the derivatives are substituted the values

$$\dot{\rho}_0=0, \qquad \dot{\sigma}_0=0,$$
$$\dot{\theta}_0=L_0 \sin\theta_0 \sin\varphi_0 \cos\varphi_0 \left(\frac{1}{I_x}-\frac{1}{I_y}\right),$$
$$\dot{\varphi}_0=L_0 \cos\theta_0 \left(\frac{1}{I_z}-\frac{1}{I_x}\sin^2\varphi_0-\frac{1}{I_y}\cos^2\varphi_0\right);$$
$$\dot{\psi}_0=L_0\left(\frac{1}{I_x}\sin^2\varphi_0+\frac{1}{I_y}\cos^2\varphi_0\right).$$

b) We will now replace, on the right sides of equations (1) to (6) $\dot{\rho}$ and $\dot{\sigma}$ by $\dot{\rho}_0$ and $\dot{\sigma}_0$, and $\dot{\theta}$, $\dot{\phi}$, $\dot{\psi}$ by

$$\left.\begin{aligned} \dot{\bar{\vartheta}}&=L \sin \vartheta \sin \varphi \cos \varphi\left(\frac{1}{I_x}-\frac{1}{I_y}\right)+\dot{\sigma}_0 \sin \rho \cos \psi-\dot{\rho}_0 \sin \psi, \\ \dot{\bar{\varphi}}&=L \cos \vartheta\left(\frac{1}{I_z}-\frac{1}{I_x} \sin^2 \varphi-\frac{1}{I_y} \cos^2 \varphi\right)+ \\ &+\frac{1}{\sin \vartheta}(\dot{\rho}_0 \cos \psi+\dot{\sigma}_0 \sin \rho \sin \psi), \\ \dot{\bar{\psi}}&=L\left(\frac{1}{I_x} \sin^2 \varphi-\frac{1}{I_y} \cos^2 \varphi\right)- \\ &-\frac{\cos \psi}{\sin \vartheta}(\dot{\rho}_0 \cos \psi+\dot{\sigma}_0 \sin \rho \sin \psi)-\dot{\sigma}_0 \cos \rho_0. \end{aligned}\right\} \quad (*)$$

Let us apply to the derived system one step of the Runge--Haine-Kutta method. Given that this step will be h (its selection is discussed below).

c) The next step of the Runge-Haine-Kutta method is applied to the system which is derived just as in b), but instead of the quantities $\dot{\rho}_0$ and $\dot{\sigma}_0$, we should use $\dot{\rho}_1$ and $\dot{\sigma}_1$ ($\dot{\rho}_1$ and $\dot{\sigma}_1$ are values of $\dot{\rho}$ and $\dot{\sigma}$ at point $t_1 = t_0 + h$, which have already been calcualted according to b)).

Now about the selection of the step h. Given a fixed step h_0. Let us solve a system with this spacing as in b). Then we will solve it with $h_0/2$ two steps according to b) and c) and again from the initial point. If the agreement is good, we take $h = h_0$; otherwise, we replace h_0 by $h_0/2$ and repeat this procedure.

Our program (to reduce computer time) was composed so the selection of spacing is only done every 50 spaces. Thus, the program operates, so to speak, with a piecewise-continuous spacing /109
with a "piece length" of 50 spaces.

Let us mention that our method of selecting the spacing virtually ensures not only sufficient accuracy of the Runge-Haine--Kutta method, but also the legitimacy of the aforementioned variations in the initial system (1)-(6).

Note: Apparently, this type of methodology can be applied to any unsolved system of differential equations of the first order. Its application in this case is justified by the fact that functions L, ρ, σ vary slowly versus ϕ and ψ for a rapidly twisted satellite.

As calculations for model problems have shown, the spacing of integration, generally speaking, increases with the passage of time. This bespeaks the fact that the deciding factor in selecting spacing is the behavior of ϕ and ψ (the satellite becomes less twisted and the spacing increases; ρ and σ will no longer be so slowly changing functions). According to the proposed program, of course, we can predict further rotation of this satellite--no longer rapidly twisted. But with too small L, the afore-noted change in system (1)-(6) may already bring about significant errors; thus the program should not be applied in the case of a slightly twisted satellite--in that case we should use the Euler equations.

4. Averaging Equations of a Nonsymmetrical Satellite

By analogy to how the equations for symmetrical satellites are written in Beletskiy's work [2], let us give the notation of the corresponding equations for a nonsymmetrical AES. We will write them with the "aerodynamic" variables θ, λ, since with these variables it is convenient to integrate and the equations themselves are more compact to write.

In writing these equations, we will take into account all factors which affect the secular evolution of rotary motion (except moments of the forces of light pressure).

The equations can be written in the same form as for the case of a symmetrical satellite, namely:

$$\begin{gathered}
\dot{\lambda}=\lambda_{\text{а}}+\lambda_{\text{д}}+\lambda_{\text{н}}+\lambda'+\lambda^{\text{дн}},\\
\dot{\theta}=\bar{\theta}_{\text{а}}+\bar{\theta}_{\text{д}}+\bar{\theta}_{\text{н}}+\theta'+\theta^{\text{дн}},\\
\dot{L}=(M_z^{\text{дн}}\cos\lambda-M_y^{\text{дп}}\sin\lambda)\sin\theta+M_x^{\text{дн}}\cos\theta-p_{\text{а}}L,\\
\frac{d\cos\theta}{dt}=f^{\text{да}}+f^{\text{дн}},\\
M^{\text{д}}_{x,\,y,\,z}=M^{\text{да}}_{x,\,y,\,z}+M^{\text{дн}}_{x,\,y,\,z},\\
P_{\text{а}}=\text{const},
\end{gathered} \tag{65}$$

where λ_a, $\bar{\theta}_a$ are aerodynamic terms, λ_d, $\bar{\theta}_d$--gravitational and part of the aerodynamics, λ', $\bar{\theta}'$ --terms governed by orbital evolution, λ_n, $\dot{\theta}_n$--magnetic terms, $M^{da}_{x,\,y,\,z}$, f^{da}--terms of aerodynamic dissipation, $M^{dn}_{x,\,y,\,z}$, f^{dn}--terms governed by eddy currents. The equations are written for the case of a circular orbit. /110

Let us discuss the part of aerodynamic and gravitational terms. The general theory of rotary satellite motion caused by perturbations which have a force function is applicable to the case of gravitational and part of the aerodynamic.

Let us note that in a circular orbit, we must take into account only the term with

$$I_3=\frac{1}{2\pi}\int_0^{2\pi}\bar{\rho}_0\frac{\sin^2\nu}{(1+\cos\nu)^2}\,d\nu,$$

since the other quantities I_i yield zeroes. Thus, let us write

$$\begin{gathered}
\lambda_{\text{а}}+\lambda_{\text{д}}=-N\left(1-\frac{3}{2}\sin^2\theta\right)\cos\theta\sin^2\lambda,\\
\bar{\theta}_{\text{а}}+\bar{\theta}_{\text{д}}=-N\left(1-\frac{3}{2}\sin^2\theta\right)\sin\theta\sin\lambda\cos\lambda,
\end{gathered}$$

where it is posited that

$$N=\frac{3}{2}\cdot\frac{\omega_0^2}{L}\left\{I_x+I_y-2I_z+3\left(\frac{2T_0I_z}{L^2}-1\right)\times\right.$$
$$\left.\times\left[I_x+(I_y-I_x)\frac{K(k)-E(k)}{k^2K(k)}\right]\right\}-\frac{\rho_a v^2 a_1^6 I_3}{2L},$$
$$k^2=\frac{(I_y-I_x)(2T_0I_z-L_0^2)}{(I_z-I_y)(L^2-2T_0I_x)},$$
$$I_z\geqslant I_y\geqslant I_x. \qquad (66)$$

Let us write the terms governed by orbital evolution (circular orbit). They will appear as:

$$\lambda'=-\dot{\Omega}(\sin i+\cos i\operatorname{ctg}\theta\sin\lambda),$$
$$\theta'=\dot{\Omega}\cos i\cos\lambda. \qquad (67)$$

Here the derivative of longitude of the ascending node is equal to

$$\dot{\Omega}=-\frac{3}{2}\cdot\frac{\breve{I}_2R_3^2\omega_0}{(R_3+h)^2}\cos i,$$
$$\breve{I}_2=(1082{,}65\pm0{,}02)\,10^{-6}. \qquad (68)$$

In order to perform averaging with respect to ϕ and ψ in magnetic perturbing moments in a circular orbit, we should make one sup- /111 position: inherent magnetic moment of the satellite and the magnetic moment induced in the soft iron of the satellite are directed along axis cz. If this condition is not fulfilled, we should use non-averaged controls. If this condition is fulfilled, the magnetic terms will appear thus:

$$\tilde{\lambda}_{\text{п}}=-\frac{I_0\mu_E}{981R^3L}\left\{\cos\vartheta\,(I_{\bar{x}}\alpha_0+I_{\bar{y}}\beta_0)+\frac{\mu_E}{R^3}k_{\text{п}}\left(1-\frac{3}{2}\sin^2\vartheta\right)\times\right.$$
$$\left.\times[(I_{\overline{xx}}-I_{\overline{zz}})\widehat{\alpha}\alpha_\theta+(I_{\overline{yy}}-I_{\overline{zz}})\widehat{\beta}\beta_0+I_{\overline{xy}}(\widehat{\alpha}\beta_\theta+\widehat{\beta}\alpha_\theta)]\right\},$$
$$\tilde{\theta}_{\text{п}}=\frac{I_0\mu_E}{981R^3L}\left\{\cos\vartheta\,(I_{\bar{x}}\alpha_\lambda+I_{\bar{y}}\beta_\lambda)+\frac{\mu_E}{R^3}k_{\text{п}}\left(1-\frac{3}{2}\sin^2\vartheta\right)\times\right.$$
$$\left.\times[(I_{\overline{xx}}-I_{\overline{zz}})\widehat{\alpha}\alpha_\lambda+(I_{\overline{yy}}-I_{\overline{zz}})\widehat{\beta}\beta_\lambda+I_{\overline{xy}}(\widehat{\alpha}\beta_\lambda+\widehat{\beta}\alpha_\lambda)]\right\}, \qquad (69)$$

where it has been designated that

$$
\left.
\begin{array}{c}
\widehat{\alpha}=\cos\theta\cos i+\sin\theta\sin\lambda\sin i, \\
\widehat{\beta}=\cos\theta\sin i-\sin\theta\sin\lambda\cos i, \\
\alpha_{\theta}=-\cos i+\operatorname{ctg}\theta\sin\lambda\sin i, \\
\beta_{\theta}=-\sin i-\operatorname{ctg}\theta\sin\lambda\cos i, \\
\alpha_{\lambda}=\cos\lambda\sin i, \quad \beta_{\lambda}=-\cos\lambda\cos i, \quad I_{\bar{x}}=-\frac{3}{2}\sin i\cos i, \\
I_{\bar{z}}=0, \quad I_{\bar{y}}=1-\frac{3}{2}\sin^2 i \quad I_{\bar{y}\bar{y}}=1-\frac{3}{2}\sin^2 i+\frac{27}{8}\sin^4 i, \\
I_{\bar{z}\bar{z}}=\frac{9}{8}\sin^2 i \quad I_{\bar{x}\bar{y}}=-\frac{3}{2}\sin i\cos i\left(1-\frac{9}{4}\sin^2 i\right), \\
I_{\bar{x}\bar{x}}=\frac{27}{8}\sin^2 i\cos^2 i; \quad \frac{\mu_E}{R^3}=0{,}315\left(\frac{R_з}{R_з+h}\right)^3, \\
k_{н}=\frac{k_{зз}}{I_0}.
\end{array}
\right\} \qquad (70)
$$

The greatest problem is in writing the terms governed by eddy currents, because we can not write averaged equations similar to (1.4), (1.5) [2]. But to take into account this factor, we can use equations (9.22). We also assume that the eddy currents lead to the formation of an additional magnetic moment, directed along axis cz. The terms governed by eddy currents can be written thus:

$$
\begin{array}{l}
\lambda^{дн}=-\frac{1}{L\sin\theta}\left(M_y^{дн}\cos\lambda+M_z^{дн}\sin\lambda\right), \\
\bar{\theta}^{дн}=\frac{1}{L}\left[\left(M_z^{дн}\cos\lambda-M_y^{дн}\sin\lambda\right)\cos\theta-M_x^{дн}\sin\theta\right],
\end{array} \qquad (71)
$$

where it is posited that /112

$$\left.\begin{aligned} M_x^{\text{дн}} &= \cos i\, M_{\bar{x}} + \sin i\, M_{\bar{y}}, \\ M_y^{\text{дн}} &= -\sin i\, M_{\bar{x}} + \cos i\, M_{\bar{y}}, \\ M_z^{\text{дн}} &= M_{\bar{z}}; \end{aligned}\right\} \tag{72}$$

$$\begin{gathered} M_{\bar{x}} = -\alpha^{\text{н}} \left[(I_{\bar{y}\bar{y}} + I_{\bar{z}\bar{z}})\, l_{\bar{x}} - I_{\bar{y}\bar{x}} l_{\bar{y}} \right], \\ M_{\bar{y}} = -\alpha^{\text{н}} \left[(I_{\bar{x}\bar{x}} + I_{\bar{z}\bar{z}})\, l_{\bar{y}} - I_{\bar{y}\bar{x}} l_{\bar{x}} \right], \\ M_{\bar{z}} = -\alpha^{\text{н}} \left[I_{\bar{y}\bar{y}} + I_{\bar{x}\bar{x}} \right] l_{\bar{z}}, \\ \alpha^{\text{н}} = \frac{k_z}{I_z} \cdot 0{,}315 \left(\frac{R_з}{R_з + h} \right)^6, \quad l_{\bar{x}} = \cos i\, l_x - \sin i\, l_y, \\ l_{\bar{y}} = \sin i\, l_x + \cos i\, l_y, \quad l_{\bar{z}} = l_z, \\ l_x = L \cos \theta, \\ l_y = -L \sin \theta \sin \lambda, \\ l_z = L \sin \theta \cos \lambda. \end{gathered} \tag{73}$$

Furthermore we will find that

$$f^{\text{дн}} = K_{\text{н}} \cos \vartheta \sin^2 \vartheta. \tag{74}$$

The term of aerodynamic dissipation, by analogy with (74), will be written thus:

$$f^{\text{да}} = K_{\text{а}} \cos \vartheta \sin^2 \vartheta. \tag{75}$$

Thus, system (1)-(6) is written with an allowance for all factors. The system was solved numerically by the Runge-Kutta method on an M-20 computer (the model problem). The results of integration are as follows: in 50 orbits of the satellite L and cos θ varied roughly by 2%, and λ and θ negligibly.

5. Methods for Defining Unknown Parameters in terms of the Results of Flight Tests

1°. Let us first formulate the problem. In equations (1)-(6), the initial values and parameters D_x^T, D_y^T, D_z^T, k_1, k_2, k_3 are known rather coarsely. We must specify them, if the values of the functions $L(t)$, $\rho(t)$, $\sigma(t)$, $\theta(t)$ at points t_1, t_2, ..., t_N are known from telemetry. Let us designate the values of these functions at point t_j by x_{ij} (i = 1, 2, 3, 4). The values of these same functions and the corresponding values to specific initial values and parameters will be designated by $\tilde{x}_{ij}$. Let us consider that dispersions of random quantities $\frac{L(t_j)}{L(t_0)}$, $\rho(t_j)$, $\sigma(t_j)$, $\theta(t_j)$ whose average values (as was shown above) are known from telemetry, are identical. We will use the method of least squares.

For the convenience of writing, let us somewhat alter the notations. Namely /113

$$x_{ij} \equiv \frac{L(t_j)}{\tilde{L}(t_0)}, \quad \tilde{x}_{ij} = \frac{\tilde{L}(t_j)}{\tilde{L}(t_0)}.$$

Let us compose the functional

$$\Phi(b_1, \ldots, b_{12}) = \sum_{i=1}^{4}\sum_{j=1}^{N}(x_{ij} - \tilde{x}_{ij})^2, \quad b_1 = D_x^T, \ldots,$$

b_1, ..., b_{12} we derive from the condition of local min Φ. This minimum will be derived by one of the gradient methods, namely the method of most rapid descent. The algorithm of the method is as follows.

Let us select the initial approximation (data are coarse values)

$$(b_1^0, \ldots, b_{12}^0) \equiv \vec{b}^0.$$

The sequence of vectors $\vec{b}^1, \vec{b}^2$, ..is calculated until $\vec{b}^n$ and $\vec{b}^{n+1}$ with the required accuracy will coincide: $\vec{b}^{n+1}$ will be the desired vector.

The sequence is calculated thus:

$$\vec{b}^{n+1} = \vec{b}^n - \lambda \operatorname{grad} \Phi (\vec{b}^n),$$

where the constant λ is such that the functional $\Phi\ (\vec{b}^n - \mu \operatorname{grad}\Phi(\vec{b}^n))$ attains its local min at $\mu = \lambda$. The method described above has been realized in a program for the case of averaged equations.

2°. Moreover, the specification of the parameters (k_x, k_y, k_z, k_1, k_2, k_3) was done by yet another method. Its essence is as follows. Given from telemetry that we know L, ρ, σ, θ, ϕ, ψ for moments of time t_1, ..., t_n. We use formulas (1)-(6) to define the perturbing moments M_1, M_2 and M_3. The derivatives of L, ρ, ...,ψ are derived with the aid of formulas of numerical differentiation.

Refinement of the desired parameters in terms of obtained values of M_1, M_2, and M_3 was done by the method of least squares.

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