INFLUENCE OF A HYPERBOLIC FLYBY OF A SMALL MASS ON THE ORBITAL EVOLUTION OF A MASSIVE BINARY

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Abstract. Method of variation of arbitrary constants is applied to determining the first order perturbations of the orbital elements of a massive close binary caused by the hyperbolic distant flyby of a small mass. The perturbations are expressed by a sequence of analytical formulas involving definite integrals of the simple type and admitting the straightforward evaluation by computer.

1. Introduction

In developing the results of (Brumberg *et al.*, 1986) this paper attempts the analytical presentation of the influence of the hyperbolic flyby of a small mass on the orbital evolution of a massive close binary. Even for this simplified case (the motion of the small particle being known, the binary motion is described by the disturbed two body problem) the analytical treatment involves difficulties and we have succeeded only in expressing the first order perturbations of the orbital elements by means of the sequence of the definite integrals of the simple type.

2. Perturbations of the Orbital Elements

The equations of the problem under consideration are as follows (Brumberg *et al.*, 1986)

$$\ddot{\mathbf{R}} = -GM\left(1 + \frac{m_3}{M}\right)\frac{\mathbf{R}}{R^3} + O\left(\frac{\rho^2}{R^2}\right), \tag{1}$$

$$\ddot{\boldsymbol{\rho}} + \frac{GM}{\rho^3}\boldsymbol{\rho} = \mathbf{F}(t, \boldsymbol{\rho}) + O\left(\frac{m_1 - m_2}{M}\frac{\rho^2}{R^2}\right) + O\left(\frac{\rho^3}{R^3}\right), \tag{2}$$

$$\mathbf{F} = \frac{Gm_3}{R^3}\left[-\boldsymbol{\rho} + \frac{3}{R^2}(\mathbf{R}\boldsymbol{\rho})\mathbf{R}\right]. \tag{3}$$

Here $\boldsymbol{\rho}$ is the position vector of the binary component m_2 with respect to the component $m_1, M = m_1 + m_2$, $\mathbf{R}$ is the position vector of the small mass $m_3 \ll M$ with respect to the centre of mass of the binary. It is assumed that for all times $\rho \ll R$, $\rho = |\boldsymbol{\rho}|$, $R = |\mathbf{R}|$. Neglecting the second degree terms with respect to ρ/R, Eqs. (1) and (2) may be separated resulting in the hyperbolic motion of m_3 and the

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disturbed two body motion of the binary components. For the hyperbolic motion of m_3 one has

$$\mathbf{R} = R(\mathbf{P} \cos V + \mathbf{Q} \sin V), \tag{4}$$

$$R \cos V = A(E - \cosh H), \qquad R \sin V = A(E^2 - 1)^{1/2} \sinh H, \tag{5}$$

$$R = A(E \cosh H - 1), \tag{6}$$

$$E \sinh H - H = Nt - \mathcal{T}, \qquad N^2 A^3 = G(M + m_3), \tag{7}$$

$\mathbf{P} = (P_1, P_2, P_3)$, $\mathbf{Q} = (Q_1, Q_2, Q_3)$ are the orthogonal vectors depending on the angular elements of the hyperbolic orbit. As initial approximation for the orbital binary motion we take the circular motion

$$\boldsymbol{\rho} = a(\mathrm{Re}(\zeta), \mathrm{Im}(\zeta), 0), \qquad \zeta = \exp(\sqrt{-1}\, nt), \qquad n^2 a^3 = GM. \tag{8}$$

The problem is to find the evolution of the binary orbit on the whole time interval $-\infty < t < \infty$ due to the hyperbolic distant ($\rho \ll R$) flyby of $m_3(-\infty < H < \infty)$. To investigate this problem an equation for determining $\rho = \rho(t)$ was derived in (Brumberg *et al.*, 1986). Here we prefer to use the classical variation of parameters method. Substituting Eqs. (8) in the right-hand sides of the equations for the osculating elements one has

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2}{n} T, \tag{9}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} + \sqrt{-1}\, e\left(\frac{\mathrm{d}\omega}{\mathrm{d}t} + \frac{\mathrm{d}\Omega}{\mathrm{d}t}\right) = \frac{1}{na}\left(2T - \sqrt{-1} S\right)\zeta, \tag{10}$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} + \sqrt{-1} \sin i \frac{\mathrm{d}\Omega}{\mathrm{d}t} = \frac{1}{na} W\zeta, \tag{11}$$

$$\frac{\mathrm{d}M_0}{\mathrm{d}t} + \frac{\mathrm{d}\omega}{\mathrm{d}t} + \frac{\mathrm{d}\Omega}{\mathrm{d}t} = -\frac{2}{na} S, \tag{12}$$

S, T, W being the components of the disturbing acceleration

$$\begin{aligned} S = \frac{1}{\rho}\boldsymbol{\rho}\mathbf{F} = \frac{Gm_3}{R^3} a\{ &-1 + \tfrac{3}{4}[P\bar{P} + Q\bar{Q} + \\ &+ \tfrac{1}{2}(P^2 + Q^2)\zeta^2 + \tfrac{1}{2}(\bar{P}^2 + \bar{Q}^2)\bar{\zeta}^2] + \\ &+ \tfrac{3}{4}[P\bar{P} - Q\bar{Q} + \tfrac{1}{2}(P^2 - Q^2)\zeta^2 + \tfrac{1}{2}(\bar{P}^2 - \bar{Q}^2)\bar{\zeta}^2]\cos 2V + \\ &+ \tfrac{3}{4}(P\bar{Q} + Q\bar{P} + PQ\zeta^2 + \bar{P}\bar{Q}\bar{\zeta}^2)\sin 2V\} \end{aligned} \tag{13}$$

$$\begin{aligned} T = \frac{1}{\rho}[\boldsymbol{\rho} \times \mathbf{F}]\mathbf{k} = \sqrt{-1}\,\frac{3}{4}\frac{Gm_3}{R^3} a\{ &\tfrac{1}{2}(P^2 + Q^2)\zeta^2 - \tfrac{1}{2}(\bar{P}^2 + \bar{Q}^2)\bar{\zeta}^2 + \\ &+ \tfrac{1}{2}[(P^2 - Q^2)\zeta^2 - (\bar{P}^2 - \bar{Q}^2)\bar{\zeta}^2] \cos 2V + \\ &+ (PQ\zeta^2 - \bar{P}\bar{Q}\bar{\zeta}^2)\sin 2V\}, \end{aligned} \tag{14}$$

$$W = \mathbf{k}\mathbf{F} = \frac{3}{4}\frac{Gm_3}{R^3}a\{(P_3 P + Q_3 Q)\zeta + (P_3 \bar{P} + Q_3 \bar{Q})\bar{\zeta} + \\ + [(P_3 P - Q_3 Q)\zeta + (P_3 \bar{P} - Q_3 \bar{Q})\bar{\zeta}]\cos 2V + \\ + [(P_3 Q + Q_3 P)\zeta + (P_3 \bar{Q} + Q_3 \bar{P})\bar{\zeta}]\sin 2V\}. \quad (15)$$

Here $\mathbf{k}$ is the unit vector orthogonal to the orbital plane of the undisturbed motion (8), $P = P_1 - \sqrt{-1}\,P_2$, $Q = Q_1 - \sqrt{-1}\,Q_2$. At the first approximation the solution of Eqs. (9)–(12) over the time interval $(-\infty, \infty)$ yields

$$\frac{\delta a}{a} = \sqrt{-1}\frac{3}{4}\frac{m_3}{M}\frac{N}{n}\{[(P^2 + Q^2)J_1(2m) \\ + (P^2 - Q^2)J_2(2m) + 2PQJ_3(2m)]w^2 - \\ - [(\bar{P}^2 + \bar{Q}^2)\bar{J}_1(2m) + (\bar{P}^2 - \bar{Q}^2)\bar{J}_2(2m) + 2\bar{P}\bar{Q}\bar{J}_3(2m)]w^{-2}\}, \quad (16)$$

$$\delta e + \sqrt{-1}e(\delta\omega + \delta\Omega) = \sqrt{-1}\,\frac{3}{4}\frac{m_3}{M}\frac{N}{n}\{[\tfrac{4}{3} - P\bar{P} - \\ - Q\bar{Q})J_1(m) - (P\bar{P} - Q\bar{Q})J_2(m) - \\ - (P\bar{Q} + Q\bar{P})J_3(m)]w - [\tfrac{3}{2}(\bar{P}^2 + \bar{Q}^2)\bar{J}_1(m) + \\ + \tfrac{3}{2}(\bar{P}^2 - \bar{Q}^2)\bar{J}_2(m) + 3\bar{P}\bar{Q}\bar{J}_3(m)]w^{-1} + \\ + [\tfrac{1}{2}(P^2 + Q^2)J_1(3m) + \tfrac{1}{2}(P^2 - Q^2)J_2(3m) + \\ + PQJ_3(3m)]w^3\}, \quad (17)$$

$$\delta i + \sqrt{-1}\sin i\,\delta\Omega = \frac{3}{4}\frac{m_3}{M}\frac{N}{n}\{[P_3 P + Q_3 Q)J_1(2m) \\ + (P_3 P - Q_3 Q)J_2(2m) + \\ + (P_3 Q + Q_3 P)J_3(2m)]w^2 + (P_3 \bar{P} + Q_3 \bar{Q})J_1(0) + \\ + (P_3 \bar{P} - Q_3 \bar{Q})J_2(0) + (P_3 \bar{Q} + Q_3 \bar{P})J_3(0)\} \quad (18)$$

$$\delta M_0 + \delta\omega + \delta\Omega = -\frac{3}{2}\frac{m_3}{M}\frac{N}{n}\{(-\tfrac{4}{3} + P\bar{P} + Q\bar{Q})J_1(0) + (P\bar{P} - Q\bar{Q})J_2(0) + \\ + (P\bar{Q} + Q\bar{P})J_3(0) + [\tfrac{1}{2}(P^2 + Q^2)J_1(2m) + \\ + \tfrac{1}{2}(P^2 - Q^2)J_2(2m) + PQJ_3(2m)]w^2 + \\ + [\tfrac{1}{2}(\bar{P}^2 + \bar{Q}^2)\bar{J}_1(2m) + \tfrac{1}{2}(\bar{P}^2 - \bar{Q}^2)\bar{J}_2(2m) + \\ + \bar{P}\bar{Q}\bar{J}_3(2m)]w^{-2}\} \quad (19)$$

with $m = n/N, w = \exp(\sqrt{-1}\,m\mathscr{T})$. The bar denotes here a conjugate quantity, for example, $\bar{J}_1(p) = J_1(p), \bar{J}_2(p) = J_2(p), \bar{J}_3(p) = -J_3(p)$.

The integrals $J_n(km)$ ($n = 1,2,3$; $k = 0,1,2,3$) are defined by the relation

$$J_n(km) = NA^3 w^{-k} \int_{-\infty}^{\infty} \frac{\zeta^k}{R^3} \begin{Bmatrix} 1 & n = 1 \\ \cos 2V, & n = 2 \\ \sin 2V, & n = 3 \end{Bmatrix} dt. \tag{20}$$

In the next section it will be shown that these integrals may be transformed to

$$J_n(p) = 2\pi \exp\left[p\left(\arcsin \frac{\sqrt{E^2 - 1}}{E} - \sqrt{E^2 - 1} \right)\right] \cdot \begin{Bmatrix} (E^2 - 1)^{-3/2} & n = 1 \\ -pE^{-2}/3, & n = 2 \\ -\sqrt{-1}\, pE^{-2}/3, & n = 3 \end{Bmatrix} + \exp(p\tfrac{1}{2}\pi) I_n(p), \tag{21}$$

$$I_n(p) = \int_0^{\infty} \exp(-pE \cosh x)[C_n(x) \cos px + S_n(x) \sin px]\, dx \tag{22}$$

with

$$C_1(x) = 2\frac{1 - E^2 \sinh^2 x}{(1 + E^2 \sinh^2 x)^2}, \qquad S_1(x) = \frac{4E \sinh x}{(1 + E^2 \sinh^2 x)^2}, \tag{23}$$

$$C_2(x) = \frac{2}{(1 + E^2 \sinh^2 x)^4} [E^4(E^2 - 2)\sinh^6 x + \\ + E^2(2E^4 - 15E^2 + 12)\sinh^4 x + \\ + (-12E^4 + 15E^2 - 2)\sinh^2 x + 2E^2 - 1], \tag{24}$$

$$S_2(x) = \frac{4E \sinh x}{(1 + E^2 \sinh^2 x)^4} [-E^2(3E^2 - 4) \sinh^4 x - \\ - 2(2E^4 - 5E^2 + 2)\sinh^2 x + 4E^2 - 3], \tag{25}$$

$$C_3(x) = \sqrt{-1}\, \frac{4E\sqrt{E^2 - 1}\, \cosh x}{(1 + E^2 \sinh^2 x)^4} \\ [E^2(E^2 - 4)\sinh^4 x - 2(3E^2 - 2)\sinh^2 x + 1], \tag{26}$$

$$S_3(x) = \sqrt{-1}\, \frac{4\sqrt{E^2 - 1}\, \sinh x \cosh x}{(1 + E^2 \sinh^2 x)^4} \\ [-E^4 \sinh^4 x - 2E^2(2E^2 - 3)\sinh^2 x + 4E^2 - 1]. \tag{27}$$

Thus, the problem is reduced to the evaluation of the integrals (22). For $p = 0$

these integrals are expressed in the elementary functions. Their evaluation for $p > 0$ involves difficulties due to the complicated behaviour of the functions under the sign of integration. Being inherent in the numerical treatment these difficulties necessitate a thorough analysis of the actual accuracy of numerical integration of the problem under consideration. For the evaluation of integrals (22) it is convenient to choose the units of measurement so that $M = a = G = 1$. Hence, $n = 1$ and the relative velocity of the binary components will be of the order of 1, $v^2 = GM/a = 1$. The velocity V of the mass m_3 in its hyperbolic motion will be

$$V^2 \approx \frac{2}{R} + \frac{1}{A}.$$

Denoting the pericentric distance of the hyperbolic orbit by $q = A(E - 1)$ we shall consider the case $q \geqslant K$, K being a large integer, $K = 10$, at least. Hence, $E \geqslant 1 + K/A$. The velocity of m_3 at the pericentre will be

$$V_q^2 \approx \frac{E+1}{E-1}\frac{1}{A}.$$

Putting $V_q = xv$ (in virtue of the hyperbolic motion of m_3 $x^2 \geqslant 2/K$) one has

$$A \approx \frac{1}{x^2}\frac{E+1}{E-1}, \qquad E \geqslant Kx^2 - 1, \qquad N \approx x^3\left(\frac{E-1}{E+1}\right)^{3/2}.$$

With assumption $x \geqslant 1$ the eccentricity E is large resulting in the estimations $A \sim x^{-2}$, $N \sim x^3$. Thus, the integrals (22) should be evaluated for $p = kx^{-3}$, $k = 1, 2, 3$, for example with $x = 1$ (a slow flyby, $p \sim k$) and $x = 10$ (a fast flyby, $p \sim 10^{-3}k$). Evaluation of the integrals (22) is considered in Section 4.

3. Transformation of Integrals $J_n(p)$

In accordance with the definition (20)

$$J_n(p) = \int_{-\infty}^{\infty} f_{np}(H)\mathrm{d}H, \tag{28}$$

$$f_{np}(H) = F_n(H)\varphi_p(H), \qquad F_n(H) = \psi_n(H)(E\cosh H - 1)^{-m_n}, \tag{29}$$

$$\varphi_p(H) = \exp[\sqrt{-1}\,p(E\sinh H - H)], \tag{30}$$

$$\psi_n(H) = \begin{cases} 1, & n = 1 \\ \frac{3}{2}E^2 - 2E\cosh H + (1 - \frac{1}{2}E^2)\cosh 2H, & n = 2 \\ (E^2 - 1)^{1/2}(2E\sinh H - \sinh 2H), & n = 3 \end{cases} \tag{31}$$

and $m_n = 2(1 + \mathrm{Entier}(n/2))$, that is $m_1 = 2$, $m_2 = m_3 = 4$. Function (29) has the poles determined by the equation

$$E \cosh H - 1 = 0. \tag{32}$$

Expressed in terms of $z = \exp H$ this equation has, in the complex plane, the roots

$$z = E^{-1}[1 \pm \sqrt{-1}(E^2 - 1)^{1/2}]$$

or

$$H = H_k = H_k^{\pm} = \sqrt{-1}\left[\pm \arcsin \frac{(E^2 - 1)^{1/2}}{E} + 2k\pi \right], \quad k = 0, \pm 1, \pm 2, \ldots \tag{33}$$

with

$$\sinh H_k = \pm \sqrt{-1}\, E^{-1}(E^2 - 1)^{1/2} \qquad \cosh H_k = E^{-1}. \tag{34}$$

Thus,

$$\varphi_p(H_k) = \exp[p(\pm \arcsin(\sqrt{E^2 - 1})/E \mp \sqrt{E^2 - 1} + 2k\pi)], \tag{35}$$

$$\psi_1(H_k) = 1, \qquad \psi_2(H_k) = \mp \sqrt{-1}\, \psi_3(H_k) = 2(E - 1/E)^2. \tag{36}$$

Using these values it is easy to find

$$\operatorname*{res}_{H = H_k} f_{np}(H) = \lim_{H \to H_k} \frac{1}{(m_n - 1)!} \frac{\mathrm{d}^{m_n - 1}}{\mathrm{d}H^{m_n - 1}} [(H - H_k)^{m_n} f_{np}(H)]$$

namely,

$$\operatorname{res} f_{1p}(H_k) = \mp \sqrt{-1}(E^2 - 1)^{-3/2} \varphi_p(H_k), \tag{37}$$

$$\operatorname{res} f_{2p}(H_k) = \tfrac{1}{3}\sqrt{-1}\, p\varphi_p(H_k)E^{-2}, \tag{38}$$

$$\operatorname{res} f_{3p}(H_k) = \mp \tfrac{1}{3} p\varphi_p(H_k)E^{-2}. \tag{39}$$

Let $ABCDA$ be the rectangular contour in the H-plane with vertices $A(-a, 0)$, $B(a, 0)$, $C(a, \sqrt{-1}\pi/2)$, $D(-a, \sqrt{-1}\pi/2)$, a being a positive number. This contour contains inside only one singular point H_0 of the function $f_{np}(H)$. The contour integral over $ABCDA$ is equal to $2\pi\sqrt{-1} \operatorname{res} f_{np}(H_0^+)$. The integral over $\overrightarrow{DC}$ will be

$$\int_{DC} f_{np}(H)\, \mathrm{d}H = \int_{-a}^{a} f_{np}(x + \sqrt{-1}\, \pi/2)\, \mathrm{d}x$$

$$= \exp(p\tfrac{1}{2}\pi) \int_{-a}^{a} \exp(-pE \cosh x)$$

$$\exp(-\sqrt{-1}\, px) F_n(x + \sqrt{-1}\tfrac{1}{2}\pi)\, \mathrm{d}x. \tag{40}$$

In the limit $a \to \infty$ the integral over $\overrightarrow{AB}$ is reduced to $J_n(p)$, the integrals over $\overrightarrow{BC}$

and $\overrightarrow{DA}$ vanish. The result is

$$J_n(p) = 2\pi\sqrt{-1}\,\mathrm{res} f_{np}(H_0^+) + \exp(p\tfrac{1}{2}\pi)I_n(p), \tag{41}$$

$$I_n(p) = \int_{-\infty}^{\infty} \exp(-pE\cosh x)\exp(-\sqrt{-1}px)F_n(x + \sqrt{-1}\tfrac{1}{2}\pi)\,\mathrm{d}x. \tag{42}$$

These expressions yield (21), (22) with

$$C_n(x) = F_n(x + \sqrt{-1}\tfrac{1}{2}\pi) + F_n(-x + \sqrt{-1}\tfrac{1}{2}\pi), \tag{43}$$

$$S_n(x) = \sqrt{-1}\,[-F_n(x + \sqrt{-1}\tfrac{1}{2}\pi) + F_n(-x + \sqrt{-1}\tfrac{1}{2}\pi)]. \tag{44}$$

4. Evaluation of Integrals $I_n(p)$

Introducing in (22) a new independent argument $t = \cosh x - 1$ and omitting parameter p in the designations let us consider

$$K_n = \delta_n E^{2m_n}\exp(\alpha)I_n(p) = \int_0^{\infty} \frac{\exp(-\alpha t)D_n(t)}{[(t+a)(t+\varepsilon)]^{m_n}}\,\mathrm{d}t \tag{45}$$

with

$$\delta_1 = \delta_2 = 1, \quad \delta_3 = -\sqrt{-1}, \quad a = 1 + \sqrt{1 - E^{-2}},$$

$$\varepsilon = 1 - \sqrt{1 - E^{-2}},\ \alpha = pE, \tag{46}$$

$$D_n(t) = C_n^* q_1(t) + pS_n^* q(t), \tag{47}$$

$$q(t) = p^{-1}\sin[p\ \mathrm{arccosh}(t+1)]. \tag{48}$$

$$q_1(t) = t^{-1/2}(t+2)^{-1/2}\cos[p\ \mathrm{arccosh}(t+1)] = \mathrm{d}q/\mathrm{d}t. \tag{49}$$

Coefficients $C_n^*(t)$, $S_n^*(t)$ related with $C_n(x)$, $S_n(x)$ by means of

$$C_n^*(t) = \delta_n E^{2m_n}(t+a)^{m_n}(t+\varepsilon)^{m_n}C_n(x), \tag{50}$$

$$S_n^*(t) = \delta_n E^{2m_n}(t+a)^{m_n}(t+\varepsilon)^{m_n}\frac{S_n(x)}{\sinh x} \tag{51}$$

are polynomials in terms of t

$$C_1^* = 2 - 4E^2 t - 2E^2 t^2, \qquad S_1^* = 4E, \tag{52}$$

$$\begin{aligned} C_2^* = {} & 2(2E^2 - 1) + 4(-12E^4 + 15E^2 - 2)t + \\ & + 2(8E^6 - 72E^4 + 63E^2 - 2)t^2 + \\ & + 8E^2(4E^4 - 19E^2 + 12)t^3 + 2E^2(14E^4 - 39E^2 + 12)t^4 + \\ & + 12E^4(E^2 - 2)t^5 + 2E^4(E^2 - 2)t^6, \end{aligned} \tag{53}$$

$$S_2^* = E[4(4E^2 - 3) - 16(2E^4 - 5E^2 + 2)t - 8(8E^4 - 13E^2 + 2)t^2 - $$
$$- 16E^2(3E^2 - 4)t^3 - 4E^2(3E^2 - 4)t^4], \tag{54}$$

$$C_3^* = \sqrt{E^2 - 1}\, E[4 - 12(4E^2 - 3)t + 8(2E^4 - 11E^2 + 6)t^2 +$$
$$+ 8(4E^4 - 19E^2 + 2)t^3 + 20E^2(E^2 - 4)t^4 + 4E^2(E^2 - 4)t^5], \tag{55}$$

$$S_3^* = \sqrt{E^2 - 1}\,[4(4E^2 - 1) - 4(8E^4 - 16E^2 + 1)t - 8E^2(8E^2 - 9)t^2 -$$
$$- 12E^2(4E^2 - 2)t^3 - 20E^4 t^4 - 4E^4 t^5]. \tag{56}$$

Using (49) and integrating by parts one has

$$K_n = \int_0^\infty \frac{\exp(-\alpha t) q(t) \mathscr{P}(t)}{[(t + a)(t + \varepsilon)]^{m_n + 1}} \mathrm{d}t, \tag{57}$$

$\mathscr{P}(t)$ being the polynomial in t

$$\mathscr{P}(t) = (t + a)(t + \varepsilon)(pS_n^* + \alpha C_n^* - (\mathrm{d}/\mathrm{d}t) C_n^*) + 2m_n(t + 1)C_n^*. \tag{58}$$

Therefore, to evaluate K_n one has to deal with the integrals of the form

$$H(k,m) = \frac{1}{p} \int_0^\infty \frac{t^k \exp(-\alpha t)}{[(t + a)(t + \varepsilon)]^m} \sin[p \operatorname{arccosh}(t + 1)] \mathrm{d}t \tag{59}$$

for $k = 0, 1, \ldots, 7$, $m = 2, 3, 4, 5$. These integrals may be easily evaluated by the known techniques (expansions in Laguerre polynomials). In terms of parameters a, ε and α $H(k, m)$ may be reduced to $H(0, 1)$

$$H(k,m) = \frac{(-1)^k}{(m!)^2} \frac{\partial^{2m+k} H(0,1)}{\partial a^m \partial \varepsilon^m \partial \alpha^k}. \tag{60}$$

$H(0,1)$ results from the linear combination of two integrals of the form

$$B = \frac{1}{p} \int_0^\infty \frac{\exp(-\alpha t)}{t + b} \sin[p \operatorname{arccosh}(t + 1)] \mathrm{d}t \tag{61}$$

with $b = a$ and $b = \varepsilon$. If B is expressed explicitly in terms of α and b then the straightforward differentiation leads to the integrals (59). This question needs further investigation.

5. Conclusion

Our aim was to investigate analytically the orbital evolution of a binary due to the distant hyperbolic flyby of a small mass. This paper does not pretend to present an exhaustive solution. The solution given here is expressed by the sequence of analytical formulas involving definite integrals of the simple type. Actual calculations may be easily performed using any literal algebraic computer system and will be reported elsewhere.

It may be added that the integrals similar to (20) may occur in other problems dealing with the superposition of elliptical and hyperbolic motions. For example, such integrals (containing, however, much simpler functions than in (20)) are found when investigating the encounter-type solutions of Hill's problem of three bodies (Henon and Petit, 1986).

References

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