Model of filaments in plasma

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Filaments with a wide-angle view

- Filaments (and the vacuum vessel) with a wide-angle view from midplane port

- Usually filaments move across the magnetic field.

- It is clear that filaments moves across the LCFS at the initial discharge phase (Ohmic).
Multi-curtain and magnetic signal

- Multi-curtain is correlated with the precursor.
  - After ELM or blob goes through near X-point, ELM or blob is almost toroidal symmetry.
  - Because the X-point has no poloidal field.
  - Two-dimensional
  - Multi-curtain

- Multi-curtain may show “ELM structure”.

Shot 117822

Hiroshima Univ.

Dα  MHD

0.130  0.132  0.134  0.136  0.138  0.140 sec

Reversed image

B.Davis
Aim

- Filament-like edge turbulences called just “filaments” are seen in many ST and tokamak plasmas for fusion experiments.
- They should be related to the energy/particle confinement.
- Therefore, it is very important to understand the unknowns such as where the filament is forming, its radial and poloidal/toroidal extent and dynamics.

- What is the origin of the filament?
  - Is it in infancy?
- What forces move the filament?
  - ExB force or JxB force
  - In experiment, a multi-curtain structure correlated to the magnetic oscillation signal.
  - This result may show the magnetic reconnection occurs by the filament. The current may exist in the filament.

- Can single fluid MHD theory treat them?
  - Single fluid MHD is very useful to treat real-sized plasmas.
Scenario of this model (Overview)

- In this model, the origin of a filament is assumed to be non-homogenous (non-uniformity) heating in the same magnetic field line or flux.

- The origin is
  - the hot region called “blob” due to the non-uniform heating
  - there are the hot and cold regions in the same magnetic field line or flux.

- Where it is forming?
  - Due to the temperature difference, thermal conduction parallel to the magnetic field by electrons occurs.
    - Also thermal conduction perpendicular to the magnetic field by ions occurs simultaneously.
  - Mainly parallel thermal conduction makes the blob into the filament-like structure.
The current penetrate in the filament from its surroundings due to the magnetic diffusion.

The change of current density makes two motions, such as:
- expand
- shrink or pinch

The magnetic diffusion and thermal conduction are the major role in the filament life.

In this model, three important factors appears:
- thermal conduction parallel to the magnetic field
- thermal conduction perpendicular to the magnetic field
- magnetic diffusion
Assumption of this model

- Initial plasma is the equilibrium state, then the non-uniform heating occurs. (e.g. Additional heating)

- The non-uniform heating process makes a “blob”.
  - e.g. Non-uniformity of NBI heating is a few % to 10 few %.
  - See the energy deposition profile of NB

- The “blob” is hotter region than that of the other region in the same magnetic field line or flux.
- The “blob” expands mainly along the magnetic field due to the thermal conduction.
Aspect ratio of a filament

- Ratio of the thermal conductivities parallel and perpendicular to the magnetic field line are as follows.
- Using Lagrange system, the equation of thermal conduction is
  \[ \frac{3n_e}{2} \frac{dT}{dt} = \lambda \nabla^2 T \]
- then
  \[ \frac{3n_e}{2} \frac{L_{//}^2}{\lambda_{//}} = \Delta t_{//} \quad \lambda_{//} = 3.16 \frac{n_e T_e \tau_e}{m_e} \]
  \[ \frac{3n_i}{2} \frac{a_f^2}{\lambda_{\perp}} = \Delta t_{\perp} \quad \lambda_{\perp} = \frac{2n_i T_i}{m_i \Omega_i \tau_i} \]
- where \( L_{//} \) is a length of the filament along the field line, \( a_f \) is a length of the filament across the field line.
- \( T_e = T_i, n_e = n_i \) are assumed due to the single fluid MHD
- Initial condition
- Generation process

- Additional heating
- Thermal conduction

Magnetic field line
Aspect ratio of a filament - continued -

- Let $\Delta t_\parallel$ equal $\Delta t_\perp$.

\[
\frac{3n_e}{2} \frac{L_\parallel^2}{\lambda_\parallel} = \frac{3n_i}{2} \frac{a_f^2}{\lambda_\perp}
\]

\[\therefore \frac{a_f}{L_\parallel} = \sqrt{\frac{\lambda_\perp}{\lambda_\parallel}}
\]

\[
\left( \frac{a_f}{L_\parallel} \right)^2 = \frac{2}{3.16}\frac{n^2}{\Omega_e\tau_e\Omega_i\tau_i} \propto \frac{n^2}{B^2T^3} \quad \left( \frac{a_f}{L_\parallel} \right) \propto \frac{n}{BT^{3/2}}
\]

- This value is very small at the edge parameters.
  - e.g. $n_e=5\text{e18m}^{-3}$, $T_e=20\text{eV}$
    The ratio is $1.66\text{e-4}$
  - Proportional to $n$, $B^{-1}$, $T^{-3/2}$

- $L_\parallel$ is not longer than $2\pi qR$, then thermal conduction time should not excess

\[
\Delta t_\parallel < \Delta t = \frac{3n_e}{2} \frac{(2\pi qR)^2}{\lambda_\parallel}
\]

- Most likely $L_\parallel$ is $\pi qR$. 
Evolution process of a filament

- Using single fluid MHD we can deduce a simple formula.

\[ \rho \frac{dV}{dt} = -\nabla p + j \times B \]

- In steady state

\[ 0 = -\nabla p_0 + j_0 \times B_0 \]

- Then the non-uniform heating occurs.

\[ \rho \frac{dV}{dt} = -\nabla (p + \tilde{p}) + (j_0 + \tilde{j}) \times (B_0 + \tilde{B}) \]

\[ \tilde{p} = n\Delta T \]
Estimation of the current density in a filament

- Magnetic diffusion time \(\rightarrow\) current penetration
- Using parallel circuit model the current density is estimated as at most (total plasma current may not change)

\[
\tilde{j} = j_0 \left( \frac{\eta_0}{\eta} - 1 \right) = \frac{3}{2} \frac{\Delta T_e}{T_e} j_0
\]

- The current in the filament makes the filament motion complicated.

\[\nabla p, \nabla B, \nabla j\]

\[j_0(r)\]

\[B \bigotimes\]

Initial condition

major R

Hot blob in the outer region

pinch

blob or filament

expand
Evaluation of the jXB force of the filament

Using the penetration current formula, we get

\[ \tilde{j} \times B_0 = \frac{3\Delta T_e}{2T_e} j_0 \times B_0 \]

\[ j_0 \times \tilde{B} = \frac{3\mu_0 a_f \Delta T_e}{4T_e} j_0^2 \]

\[ \tilde{B} \approx \frac{\mu_0 \tilde{j} \pi a_f^2}{2 \pi a_f} = \frac{\mu_0 a_f \tilde{j}}{2} \approx \frac{3\mu_0 a_f \Delta T_e}{4T_e} j_0 \]

\[ \tilde{j} \times \tilde{B} = \frac{9\mu_0 a_f}{8_e} \left( \frac{\Delta T_e}{T} \right)^2 j_0^2 \]

Thus, \( \tilde{j} \times B_0 \) is always largest.

\[ \tilde{j} \times B_0 \gg j_0 \times \tilde{B} \gg \tilde{j} \times \tilde{B} \]
Evaluation of the $j \times B$ force of the filament -continued.

- **Comparison with pressure gradient**

\[
\nabla \tilde{p} \sim \frac{\Delta T_e}{T_e} \nabla p_0 = \frac{\Delta T_e}{T_e} j_0 \times B_0
\]

\[
\tilde{j} \times B_0 = \frac{3\Delta T_e}{2T_e} j_0 \times B_0
\]

- **Thus,** $\tilde{j} \times B_0 > \nabla \tilde{p}$

- $j \times B$ force due to the penetration current expand a part of the blob and also pinch the other part.

- Hot region $\Rightarrow$ expanding (low density region) + pinch (high density region)
- Generation process:
  - thermal conduction
  - magnetic field line

- Evolution process:
  - magnetic diffusion
  - pinch
  - expand
Movement depend on the initial figure of a “blob”

- In general a figure of blob is not spherical nor cylindrical symmetry.
- Therefore, the movement due to the penetration current may be more complicated.

The rotation may occur due to the conservation of momentum
Gas Puff Imaging (GPI) provides various motions of filaments
magnetic diffusion vs. thermal conduction

- Using the induction equation of the magnetic field $B$

- **magnetic diffusion time** is
  \[ \tau_{md} = \frac{\mu}{\eta} a_f^2 \]

- **thermal conduction time** is
  \[ \Delta t_{\perp} = \frac{3 n_i}{2} \frac{a_f^2}{\lambda_{\perp}} \]

- Therefore, if these times are equal, the generation rate of the filament may be the maximum.
Birth location of a filament

- Let the magnetic diffusion time equal the thermal conduction time across the magnetic field to deduce the birth position of the filament.

\[ \tau_{md} = \Delta t_\perp \]
\[ \frac{\mu_0 a_f^2}{\eta_f} = \frac{3 n_f a_f^2}{2 \lambda_\perp} \]
\[ \beta_f = \frac{n_f T_f}{B_f^2 \sqrt{2 \mu_0}} = \frac{3}{4} \sqrt{\frac{m_e}{m_i}} \]

- In H plasma, \( \beta_f = 0.0124 \)
- In D plasma, \( \beta_f = 0.00875 \)

- Therefore, the birth position of the filament is the edge of ST and/or tokamak plasmas.
Evaluation of the filament velocity $V_{\text{perp}}$

- Using the equation of motion we get

$$\rho \frac{dV}{dt} = O(\vec{j} \times \vec{B}_0)$$

$$V \approx \frac{\Delta T_e j_0 B_0}{8 \rho T_e} \tau_{md} = \frac{\Delta T_e p_0 \tau_{md}}{8 T_e n_i m_i L_p} = \frac{\Delta T_e \tau_{md}}{8 m_i L_p}$$

- Therefore, the filament moves rapidly with the increase temperature difference $\Delta T_e$.

- This velocity should be related to the filament generation rate, because the filament escape the heating region.
Scaling law using this model

- The energy in a filament is estimated at

\[ \Delta W_f = n_f T_f \pi a_f^2 L_{||} \]

- If the filament goes out the plasma within the magnetic diffusion time, then the power loss of the filament is about

\[ P_{out,fil} = \frac{1}{2} \frac{\Delta W_f}{\tau_{md}} = \frac{n_f T_f \pi a_f^2 L_{||}}{2 \mu_0 a_f^2 / \eta_f} = \frac{n_f T_f \eta_f \pi L_{||}}{2 \mu_0} \]

where numerical factor \( \frac{1}{2} \) is due to the random walk of the filament.

- In general the energy confinement time is defined as follows.

\[ 0 = P_{input} - \frac{W}{\tau_E} = P_{input} - W \left( \frac{1}{\tau_{E,NC}} + \frac{1}{\tau_{E,fil}} + \cdots \right) \]
In steady state, the power loss of the filaments may equal the total input power
\[ P_{\text{input}} = N \times P_{\text{out,fil}} \]
where \( N \) is related to the generation rate of the filaments.
\( N \) may be related to the input power, because the generation rate of the filament depends on the temperature rise of the blob and the non-uniform efficiency.
\[ N \propto \left( \xi_{\text{OH}} P_{\text{OH}} + \xi_{\text{NBI}} P_{\text{NBI}} + \cdots \right) \]
where each \( \xi \) is the non-uniform efficiency of each heating method, respectively. In general these efficiencies are not equal.
\[ \xi_{\text{OH}} < \xi_{\text{NBI}} \quad \text{In general} \quad \xi_{\text{OH}} < \xi_{\text{additonal heating}} \]
Then, only OH heating case \( N \) is proportional to \( P_{\text{total}} \).
In general
\[ N \propto P_{\text{total}}^\alpha, \quad 0 < \alpha \leq 1 \]
Scaling law using this model - continued.

- At last the energy confinement time is estimated as

\[ \tau_{E,fil} = \frac{W}{N \times P_{out,fil}} = \frac{W}{N \frac{n_f T_f \eta_f \pi L_{rl}}{2 \mu_0}} \]

- Let \( L_{rl} = \pi R_q \) and \( B_f \approx B_t \) in above equation, and use

\[ \beta_f = \frac{n_f T_f}{B_f^2} = \frac{3}{4} \sqrt{\frac{m_e}{m_i}} \]

\[ \tau_{E,fil} \propto \frac{\langle nT \rangle I_p V}{N n_f 1.5} \propto \frac{\langle nT \rangle I_p V}{P^\alpha n_f 1.5}, \quad 0 < \alpha \leq 1 \]

- It seems L-mode scaling, even though this is not a dimensionless formula yet.

\[ \tau_E = \frac{1}{B} F \left( \frac{T}{\alpha^2 B^2} \right) \]
Conclusion

- A filament model using simple fluid MHD is proposed.
- In this model a “blob” mainly expand along the magnetic field line, and the blob becomes a filament-like shape.
- That is called the “filament”.
- In this model, the generation and extinction processes of the filament are decided by the magnetic diffusion and thermal conduction.
- According this model the scaling of the energy confinement time is also estimated.
- The scaling obtained using this model is similar to the L-mode scaling.
Further problem of this model

- What condition determine the generation rate of filaments?

- Can we get completely L-mode scaling?
  - dimensionless formula

- What is the L-H transition and H-mode?

- Can we get cheap nuclear fusion reactor?
  - It is further question
GPI Diagnostic setup in NSTX

- Use re-entrant port and linear gas manifold.
- Use **He**, D$_2$, or Ar puffs.
- Use beam-splitter and PMTs (100 kHz bandwidth) for discrete fast chords.
Typical power deposition profile of NB in NSTX

R. Bell
Interpretation of multi-curtain

- Multi-curtain may be the magnetic surfaces, which are dragged by ELMs or blobs.
  - After ELM or blob goes through near X-point, ELM or blob is almost toroidal symmetry.
  - Because the X-point has no poloidal field.
  - multi-curtain
  - Multi-curtain may show “ELM structure”.

**Photron**

- 70000 fps
- Start

**FASTCAM-APX RS 2**

- 1/245000 sec
- frame : 9101
- 128 x 128
- +00:00:00.130000sec