Determination of eddy currents in the vacuum vessel of spherical tokamaks

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Introduction

The distribution of currents induced in the vacuum vessel of spherical tokamaks is required for startup simulations, equilibrium reconstruction and control.

Recently [1,2], a method has been developed that accurately describes the local and non-local coupling effects between diverse regions of the vacuum vessel and external sources.

The method reduces the integro-differential equation that governs the evolution of the surface current density induced in a thin axisymmetric shell to a circuit model.

In this paper, the model is improved and calculations of the electromotive force and induced current distribution on the vessel wall are compared with measurements in the ETE spherical tokamak.

Poloidal cross-section of ETE

- Crown
- Truss
- Elongation coil
- Internal compensation coil
- Ohmic solenoid
- Plasma
- External compensation coil
- Ring
- TF coil
- Equilibrium coil
- Vacuum vessel

(1m x 1m)
**ETE vacuum vessel**

Three shells of different thicknesses joined at $\theta_1$ and $\theta_2$ and their conjugate angles: outer cylinder $\delta_1=4.8\text{mm}$, torispherical head $\delta_2=6.35\text{mm}$ and inner cylinder $\delta_3=1.2\text{mm}$

Location of loop voltage sensors shown in red
**Electrodynamics formulation**

Faraday’s law relating the toroidal surface current density to the poloidal flux function in a thin axisymmetric shell with local values of the conductivity and thickness

\[ K_T (\theta, t) = - \frac{\sigma_s \delta_s}{2\pi h_\zeta (\theta)} \frac{\partial \Phi_P (\theta, t)}{\partial t} \]

Integro-differential equation that governs the surface current density evolution

\[ \frac{2\pi h_\zeta (\theta)}{\sigma_s \delta_s} K_T (\theta, t) = -\mu_0 \int_0^{2\pi} \frac{\partial K_T (\theta', t)}{\partial t} G (\theta, \theta') h_\theta (\theta') d\theta' - \frac{\partial \Phi_{P,ext} (\theta, t)}{\partial t} \]

where \( G(\theta,\theta') \) is the Green’s function for the axisymmetric Ampère’s law

Total toroidal current induced in the vacuum vessel

\[ I_T (t) = \int_0^{2\pi} K_T (\theta, t) h_\theta (\theta) d\theta \]

\( h_\zeta(\theta), h_\theta(\theta) \) – scale factors of the spectral representation for the vessel centerline
Surface current density

Fourier series representation in one sector of the vessel wall (torispherical head)

\[ \theta_1 < \theta < \theta_2 : \quad K_T^{(2)}(\theta, t) = \frac{1}{2\pi h_\theta(\theta)} \sum_{n=0}^{\infty} I_n^{(2)}(t) \cos \left( \frac{n\pi}{\theta_2 - \theta_1} (\theta - \theta_1) \right) \]

Total current in the vacuum vessel

\[ I_T(t) = \frac{\theta_1}{\pi} I_0^{(1)}(t) + \frac{\theta_2 - \theta_1}{\pi} I_0^{(2)}(t) + \frac{\pi - \theta_2}{\pi} I_0^{(3)}(t) \]

Jump conditions for continuous electromotive force

\[
\frac{K_T^{(1)}(\theta_1, t)}{\sigma_1 \delta_1} = \frac{K_T^{(2)}(\theta_1, t)}{\sigma_2 \delta_2}, \quad \frac{K_T^{(2)}(\theta_2, t)}{\sigma_2 \delta_2} = \frac{K_T^{(3)}(\theta_2, t)}{\sigma_3 \delta_3}
\]

used to verify accuracy of the solution
Circuit equations for vacuum vessel

Substituting the Fourier expansions in the equation that governs the evolution of \( K_T(\theta,t) \) and taking moments, one obtains a truncated set of \( 3(\ell+1) \) circuit equations for the Fourier coefficients \( I^{(1)}_n(t) \), \( I^{(2)}_n(t) \) and \( I^{(3)}_n(t) \) with \( m=0,1,2\ldots \ell \) and \( n=0,1,2\ldots \ell \)

For example, the equation for \( I^{(2)}_n(t) \) is

\[
\sum_{n=0}^{\ell} \left( R^{(2)}_{mn} I^{(2)}_n(t) + L^{(2)}_{mn} \frac{dI^{(2)}_n(t)}{dt} + M^{(2,1)}_{mn} \frac{dI^{(1)}_n(t)}{dt} + M^{(2,3)}_{mn} \frac{dI^{(3)}_n(t)}{dt} \right) = -\frac{d}{dt} \left( \frac{1}{\pi} \int_{\theta_1}^{\theta_2} \Phi_{P,ext}(\theta,t) \cos \frac{m\pi (\theta - \theta_1)}{\theta_2 - \theta_1} d\theta \right)
\]

with similar equations for \( I^{(1)}_n(t) \) and \( I^{(3)}_n(t) \)

Resistance coefficients for the torispherical head sector

\[
R^{(2)}_{mn} = \frac{1}{\pi \sigma_2 \delta_2} \int_{\theta_1}^{\theta_2} \frac{h_\zeta(\theta)}{h_\theta(\theta)} \cos \frac{m\pi (\theta - \theta_1)}{\theta_2 - \theta_1} \cos \frac{n\pi (\theta - \theta_1)}{\theta_2 - \theta_1} d\theta
\]
Inductance coefficients

Self-inductance coefficients for the torispherical head sector ($\varepsilon \ll 1$)

\[
L_{mn}^{(2)} = \frac{\mu_0}{2\pi^2} \int_{\theta_1}^{\theta_2} \left[ \left( \int_{\theta_1}^{\theta - \varepsilon} + \int_{\theta + \varepsilon}^{\theta_2} \right) G(\theta, \theta') + \int_{\theta_1}^{\theta_2} G(\theta, -\theta') \right] \cos \frac{m\pi (\theta - \theta_1)}{\theta_2 - \theta_1} \cos \frac{n\pi (\theta' - \theta_1)}{\theta_2 - \theta_1} d\theta' d\theta 
+ \frac{\mu_0 \varepsilon}{\pi^2} \int_{\theta_1}^{\theta_2} h_\zeta(\theta) \left[ \ln \left( \frac{8h_\zeta(\theta)}{\varepsilon h_\theta(\theta)} \right) - 1 \right] \cos \frac{m\pi (\theta - \theta_1)}{\theta_2 - \theta_1} \cos \frac{n\pi (\theta - \theta_1)}{\theta_2 - \theta_1} d\theta
\]

Mutual inductance coefficients between outer cylindrical wall and torispherical head

\[
M_{mn}^{(1,2)} = \frac{\mu_0}{2\pi^2} \int_{0}^{\theta_1} \int_{\theta_1}^{\theta_2} \left[ G(\theta, \theta') + G(\theta, -\theta') \right] \cos \frac{m\pi \theta}{\theta_1} \cos \frac{n\pi (\theta' - \theta_1)}{\theta_2 - \theta_1} d\theta' d\theta
\]

Symmetry properties

\[
R_{mn}^{(1)} = R_{nm}^{(1)}, \quad R_{mn}^{(2)} = R_{nm}^{(2)}, \quad R_{mn}^{(3)} = R_{nm}^{(3)}, \\
L_{mn}^{(1)} = L_{nm}^{(1)}, \quad L_{mn}^{(2)} = L_{nm}^{(2)}, \quad L_{mn}^{(3)} = L_{nm}^{(3)}, \\
M_{mn}^{(2,1)} = M_{nm}^{(1,2)}, \quad M_{mn}^{(3,1)} = M_{nm}^{(1,3)}, \quad M_{mn}^{(3,2)} = M_{nm}^{(2,3)}
\]
External sources

Right-hand side of the circuit equations in terms of the currents in external sources

\[- \frac{d}{dt} \left( \frac{1}{\pi} \int_{\theta_1}^{\theta_2} \Phi_{P, ext}(\theta, t) \cos \frac{m\pi}{\theta_2 - \theta_1} d\theta \right)\]

\[= -M^{(2)}_{m,\Omega} \frac{dI_{\Omega}}{dt} - \sum_{n=1}^{\infty} M^{(2)}_{m,\Omega n} \frac{dI_n}{dt} - M^{(2)}_{m,TF} \frac{dI_{TF}(t)}{dt} - \sum_{k} M^{(2)}_{m,k} \frac{dI_k(t)}{dt}\]

Mutual inductances between the Fourier component of order \(m\) in shell \((s)\) and:

- \(M^{(s)}_{m,\Omega}\) – solenoids and coils that form the OH system
- \(M^{(s)}_{m,\Omega n}\) – proximity effect current components in the OH system
- \(M^{(s)}_{m,TF}\) – eddy currents in the central column of the toroidal field coil
- \(M^{(s)}_{m,k}\) – additional external coils (equilibrium coils)
Circuit equations for external sources

Voltage drop in the OH system

\[ v_\Omega(t) = R_\Omega I_\Omega(t) + L_\Omega \frac{dI_\Omega}{dt} + \sum_{n=1}^{\infty} L_n \frac{dI_n}{dt} + \frac{N_\Omega \ell_{eff}}{h_\Omega} \frac{d\Phi_{TF}}{dt} \]

\[ + \sum_{m=0}^{\ell} \left( M_{\Omega,m}^{(1)} \frac{dI_m^{(1)}(t)}{dt} + M_{\Omega,m}^{(2)} \frac{dI_m^{(2)}(t)}{dt} + M_{\Omega,m}^{(3)} \frac{dI_m^{(3)}(t)}{dt} \right) + \sum_k M_{\Omega,k} \frac{dI_k(t)}{dt} \]

\( R_\Omega, L_\Omega \) – resistance and inductance of the OH system, neglecting eddy current and proximity effects

\( L_n \) – internal inductance associated with the proximity effect current components \( I_n(t) \)

\( \Phi_{TF}(t) \) – flux associated with currents induced in the central column of the TF coil

\( M_{\Omega,m}^{(s)} \) – mutual inductance between the Fourier component \( I_m^{(s)}(t) \) of order \( m \) in shell \( s \) and the OH system

\( M_{\Omega,k} \) – mutual inductance between coils carrying current \( I_k(t) \) and the OH system

\( N_\Omega, h_\Omega \) – number of turns and height of the OH solenoid

\( \ell_{eff} \) – effective length of the OH solenoid in series with the internal compensation coils
The model was used to evaluate the current induced in the vacuum vessel wall of ETE, and to calculate the loop voltage produced by the external sources and eddy currents, including vessel, central column and proximity effect components.
Electromotive force on the vacuum vessel wall

Loop voltage measurements (points) and simulations (lines)
Current distribution test shot

The improved model was used also to calculate the current distribution on the vessel wall, previously presented in [1]. The passive equilibrium coils were mistakenly included in the previous calculation, causing a time lag apparent in some signals.

Current in the OH circuit (left) and total current induced in the vessel wall (right) for the current distribution test shot: points – measurements, continuous lines – simulations.
Current distribution on the vessel wall measured on subsequent shots (8% error) using a removable Rogowski coil
Conclusions

A new circuit model for the current distribution induced on the vacuum vessel wall of spherical tokamaks, including central column and proximity effect contributions, was developed and successfully tested, particularly for loop voltage simulations in ETE.

The circuit equations can be used in two ways:

1. dividing the wall in a small number of sectors with a relatively large number of Fourier coefficients ($\ell=4$) for the current in each sector, as shown in this paper, or
2. dividing the wall in a large number of sectors with a small number of coefficients. In this case, taking $\ell=0$ corresponds to dividing the wall in a large number of rings with uniform current density, similarly to the model adopted in previous works [3,4,5], though with a precise definition of the coefficients in the circuit model.

Next, the plasma will be included in the model for startup simulations.