International Trade and Agglomeration in Asymmetric World: Core-Periphery Approach

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Spatial economy aspects can’t be explained by Perfect Competitive Market Model. Pointed out by R. Mundell (1957), formally proved by D. Starrett (1978) in Starrett’s impossibility theorem: Consider an economy with a finite number of locations. If space is homogeneous, transport is costly, and preferences are locally nonsatiated, then there exists no competitive equilibrium involving the transport of goods between locations.


Two-sectoral economy: M-sector and A-sector.

Two types of labor: industrial (mobile in long run) and agricultural (immobile)

A-sector (Perfect Competition, PC): CRS, Homogeneous Good and Zero Transportation Costs

M-sector (Monopolistic Competition, MC): Dixit-Stiglitz-type production, Variety of Goods, Iceberg-type Transportation Costs

Two regions: “Home” and “Foreign”, consumer’s preferences are identical for both countries and both types of workers.

Utility function $U = C_M^\mu \cdot C_A^{1-\mu}$, $C_M = \left( \frac{N}{\int_0^s c(s)^\rho ds} \right)^{\frac{1}{\rho}}$,

$0 < \mu < 1$, $\rho = \frac{\sigma - 1}{\sigma}$, $\sigma > 1$ — elasticity of substitution, mass of firms $N$
# Short Run and Long Run Equilibria

## Short Run Equilibrium (International Trade)
- Industrial labor supposed to be immobile in short run;
- Perfect Competitive equilibrium in A-sector;
- Monopolistic-competitive equilibrium in M-sector;
- Consumption equilibrium over all markets.

## Long Run Equilibrium (Agglomeration)
- Industrial labor flows into the region with higher Welfare
- In Long Run Equilibrium labor shares should be persistent
- Short Run Equilibrium conditions (except labor immobility)
Notions and Definitions (Exogenous)

- $0 < \mu < 1$ – expenditure share of M-sector goods: $\mu = 0$ purely PC-economy, $\mu = 1$ purely MC-economy
- $0 < \rho = \frac{\sigma - 1}{\sigma} < 1$ – CES-function parameter, $1 - \rho = \frac{1}{\sigma}$ — consumer’s “taste for variety”
- M-sector: $f > 0$ fixed costs, $m$ marginal costs of labor, $\tau > 1$ Iceberg-type transportation costs: to ship the unit of good we should produce $\tau > 1$ units, as if $\tau - 1$ units “melt” in transit of one unit of the good, $\varphi = \tau^{1-\sigma} \in (0,1)$ – “trade freeness”
- A-sector: fixed costs $f_a = 0$, marginal costs of agricultural labor $m_a = 1$ (normalization), transportation is costless $\tau_a = 1$
- $0 < \theta < 1$ – “Home” share of immobile (agricultural) labor, $1 - \theta$ is “Foreign” share, W.L.o.G. assume $\theta \geq 1/2$, Symmetry Assumption: $\theta = 1/2$
- In Short Run only: $0 \leq \lambda \leq 1$ – “Home” share of industrial labor (immobile in short run), $1 - \lambda$ is “Foreign”, $\lambda = 0$ and $\lambda = 1$ are total agglomeration cases
In Long Run: \( 0 \leq \lambda \leq 1 \) – equilibrium “Home” share of mobile (industrial) labor

Equilibrium prices: \( p_{HH} \) – Home-produced and Home-consumed, \( p_{HF} \) – Home-produced and Foreign-consumed, \( p_{FH}, p_{FF} \) are defined analogously

Equilibrium agricultural wage: in fact, \( w_a = 1 \) for both countries, due to perfect competition and costless transportation

Equilibrium industrial wages: \( w_H \) — Home wage, \( w_F \) — Foreign wage

Equilibrium firm’s masses \( N_H \) and \( N_F \) under Free Entry Assumption
Traditional point of view: agglomeration, or centripetal, forces (‘market access effect’ and ‘cost of living effect’) struggle against dispersion, or centrifugal ones (‘market crowding effect’)

Two proving grounds with clear outcome:
- Perfect competition: resultant force is centrifugal
- Monopolistic competition: resultant force is centripetal

So in fact: monopolistic competition struggles against perfect one!

- In CP model natural correlation of forces’ measure is expenditure share $\mu$
- $\mu = 1$ — pure monopolistic competition, $\mu = 0$ — pure perfect competition
In the World of Symmetry ($\theta = 1/2$)

**Short Run Equilibrium:** Existence and Uniqueness analytically proved by P. Mossay (2006)

**Long Run Equilibria:**
F. Robert-Nicoud (2005)

**Tomahawk diagram** (taken from Baldwin et al. (2003))

\[ \varphi = \tau^{1-\sigma} \in (0,1) \] — trade freeness,
\[ s_n = \lambda \] — the Home share of industrial labor
Baldwin et al. (2003), Economic Geography and Public Policy: “...intense intractability of the CP model means that numerical simulation ... is the only way forward.”

Numerical simulations: ”Broken Tomahawk” for $\theta < 1/2$, taken from Baldwin et al. (2003). Sustain points and break point can be in various relative positions.

Berliant and Kung (2009): “…conclusions drawn from the use of this bifurcation to generate a core-periphery pattern are not robust. Generically, this class of bifurcations is a myth, an urban legend.”
Theorem

Asymmetric CP model is not intractable! Numerical simulation is useful but not the only way forward.

Theorem

Though bifurcation is a myth, the most of conclusions are robust under moderate asymmetry. Changes are considerable only under sufficiently large asymmetry.

Theorem

Asymmetry even simplifies equilibrium structure.
Asymmetric CP model is not intractable! Numerical simulation is useful but not the only way forward.

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Asymmetry even simplifies equilibrium structure.
Guys, You’re Wrong!

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Short Run Equilibrium

\[ p_{HH} = \frac{w_H \cdot m \cdot \sigma}{\sigma - 1}, \quad p_{HF} = \frac{w_H \cdot \tau \cdot m \cdot \sigma}{\sigma - 1}, \]
\[ p_{FH} = \frac{w_F \cdot m \cdot \tau \cdot \sigma}{\sigma - 1}, \quad p_{FF} = \frac{w_F \cdot m \cdot \sigma}{\sigma - 1}, \]
\[ N_H = \frac{\lambda L}{f \cdot \sigma}, \quad N_F = \frac{(1 - \lambda) L}{f \cdot \sigma} \]

Moreover

\[ (1 - \mu) (w_H \cdot \lambda \cdot L + w_F \cdot (1 - \lambda) \cdot L + 1 \cdot L_a) = 1 \cdot L_a \]

\[ \iff w_H = \frac{\mu}{1 - \mu} \cdot \frac{L_a \cdot \frac{w_H}{w_F}}{L \left( \lambda \frac{w_H}{w_F} + (1 - \lambda) \right)}, \quad w_F = w_H \cdot \frac{w_F}{w_H} \]
It is sufficient to find **equilibrium relative wage** \( \frac{w_H}{w_F} \), as a root of equation:

\[
(1 - ((1 - \theta) + \mu \theta)(1 - \varphi^2)) \cdot \left( \frac{w_H}{w_F} \right)^{\sigma - 1} - \varphi \cdot \left( \frac{w_H}{w_F} \right)^{2\sigma - 1} = \frac{\lambda}{1 - \lambda}
\]

(1 - ((1 - \mu) \theta + \mu)(1 - \varphi^2)) \cdot \left( \frac{w_H}{w_F} \right)^{\sigma} - \varphi

Is it intractable?
Not at all!

**Existence and Uniqueness (generalization of P.Mossay (2006)):**

There exists the unique positive root of this equation:

\[
\varphi^{\frac{1}{\sigma}} < \frac{w_H}{w_F} < \varphi^{-\frac{1}{\sigma}}.
\]
It is sufficient to find *equilibrium relative wage* \( \frac{w_H}{w_F} \), as a root of equation:

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(1 - ((1 - \mu) \theta + \mu)(1 - \varphi^2)) \cdot \left( \frac{w_H}{w_F} \right)^{\sigma} - \varphi
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Explicit Function Technique

Relative wage is *implicit function* of all exogeneous parameters \( \frac{w_H}{w_F} = x(\theta, \lambda, \varphi, \mu, \sigma) \) defined by equation \( F(x; \theta, \lambda, \varphi, \mu \sigma) = 0 \).

Comparative statics of relative wage w.r.t. \( \pi \in \{\theta, \lambda, \varphi, \mu, \sigma\} \):

\[
\frac{\partial x}{\partial \pi}(\theta, \lambda, \varphi, \mu, \sigma) = \left( \frac{\partial F}{\partial \pi} \right) \left( - \frac{\partial F}{\partial x} \right)
\]

\[
w_H(\theta, \lambda, \varphi, \mu \sigma) = \frac{\mu}{1 - \mu} \cdot \frac{L_a \cdot x(\theta, \lambda, \varphi, \mu, \sigma)}{L(\lambda \cdot x(\theta, \lambda, \varphi, \mu, \sigma) + (1 - \lambda))}
\]

and so on.
Comparative Statics of Relative Wage

Relative wage $\frac{w_H}{w_F}$

- increases w.r.t. agricultural labor share $\theta$
- is monotone w.r.t. industrial labor share $\lambda$. Type of monotonicity depends on parameter’s relation
  
  
  * increases when $\varphi > \sqrt{1 - \frac{1 - \varphi^2}{(1 + \mu)(1 - \mu)^2 - \alpha^2}} \iff \mu > \frac{\sqrt{1 - (1 - \varphi^2)\alpha^2} - \varphi}{\sqrt{1 - (1 - \varphi^2)\alpha^2 + \varphi}}$
  
  * decreases when $\varphi < \sqrt{1 - \frac{1 - \varphi^2}{(1 + \mu)(1 - \mu)^2 - \alpha^2}} \iff \mu < \frac{\sqrt{1 - (1 - \varphi^2)\alpha^2} - \varphi}{\sqrt{1 - (1 - \varphi^2)\alpha^2 + \varphi}}$

  where $\alpha = 2\theta - 1 = \theta - (1 - \theta)$ – relative measure of asymmetry in agricultural labor (NB: $\theta \geq 1/2$!)
Consumer’s *price indices* (PI) for both regions

\[
P_H = \left( \int_0^{N_H} p_{HH}^{1-\sigma}(j) \, dj + \int_0^{N_F} p_{FH}^{1-\sigma}(j) \,dj \right)^{\frac{1}{1-\sigma}}
\]

\[
P_F = \left( \int_0^{N_H} p_{HF}^{1-\sigma}(j) \, dj + \int_0^{N_F} p_{FF}^{1-\sigma}(j) \,dj \right)^{\frac{1}{1-\sigma}}
\]

Moreover, *agricultural consumer welfares* \( V^a_H = \frac{w^a_H}{(P_H)^\mu} = \frac{1}{(P_H)^\mu} \),

\( V^a_F = \frac{w^a_F}{(P_F)^\mu} = \frac{1}{(P_F)^\mu} \), because in equilibrium \( w^a_H = w^a_F = 1 \)
### Comparative Statics of Price Indices

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Comparative Statics of Price Indices

**Price indices**

- Home \( \text{CPI} \) \( P_H \) increases w.r.t agricultural labor share \( \theta \), while foreign \( P_F \) decreases, thus relative price index (RPI) \( \frac{P_H}{P_F} \) increases w.r.t. \( \theta \)
- RPI \( \frac{P_H}{P_F} \) decreases w.r.t. industrial labor share \( \lambda \)

**Agricultural welfare**

- Home agricultural welfare \( V_H^a \) decreases w.r.t agricultural labor share \( \theta \), while foreign \( V_F^a \) increases, thus relative welfare \( \frac{V_H^a}{V_F^a} \) decreases w.r.t. \( \theta \)
- Relative agricultural welfare \( \frac{V_H^a}{V_F^a} \) increases w.r.t. industrial labor share \( \lambda \)
Welfare (equilibrium utility) for industrial workers $V_H = \frac{w_H}{(P_H)^{\mu}}$,

$V_F = \frac{w_F}{(P_F)^{\mu}}$, relative welfare $\frac{V_H}{V_F} = \frac{w_H}{w_F} \cdot \left(\frac{P_F}{P_H}\right)^{\mu}$

- $V_H$ and $V_H/V_F$ increase w.r.t. agricultural labor share $\theta$

- Let $\mu \geq \rho$ (Black Hole), then relative welfare $\frac{V_H}{V_F}$ increases w.r.t. industrial labor share $\lambda$

- Let $\mu < \rho$, then relative welfare $\frac{V_H}{V_F}$ increases w.r.t. industrial labor share $\lambda$ for $\phi \geq \phi^B = \frac{\rho - \mu}{\rho + \mu} \sqrt{\frac{1 - \alpha^2}{\left(\frac{1 + \mu}{1 - \mu}\right)^2 - \alpha^2}}$, where

$$\alpha = 2\theta - 1$$

- Let $\mu < \rho$ and $\phi < \phi^B$, then derivative $\frac{\partial}{\partial \lambda} \left(\frac{V_H}{V_F}\right)(0) > 0$ and changes sign not more than twice for $\lambda \in (0, 1)$
Patterns of Relative Welfare Change

\[ \theta = 0.55, \mu = 0.2, \rho = 0.25 \]
Long Run Equilibria

Ad hoc dynamics:

\[
\dot{\lambda} = \lambda \cdot \left[ V_H(\lambda) - (\lambda \cdot V_H(\lambda) + (1 - \lambda) V_F(\lambda)) \right]
\]
or the same

\[
\dot{\lambda} = \lambda \cdot (1 - \lambda) \cdot \left[ V_H(\lambda) - V_F(\lambda) \right]
\]

Agglomerated equilibria:
\[\lambda^0 = 1 \text{ (Core)} \text{ or } \lambda^0 = 0 \text{ (Periphery)}\]

Interior equilibria: \[V_H(\lambda^0) = V_F(\lambda^0), \ 0 < \lambda^0 < 1.\]

Main questions:
- Stability of agglomerated equilibria
- Number and stability of interior equilibria
There is no need of any differential equation.
It Is Sufficient to Know:

- \( \frac{V_H}{V_F}(0) < 1 \iff \phi < \phi_0^S \), \( \frac{V_H}{V_F}(1) > 1 \iff \phi > \phi_1^S \) below or above "sea level", sustain points \( \phi_0^S \), \( \phi_1^S \) are the smallest roots of equation

\[
\theta(1 - \mu)\phi^{\frac{\mu - \rho}{\rho}} + (1 - \theta(1 - \mu))\phi^{\frac{\mu + \rho}{\rho}} = 1
\]

\[
(1 - \theta)(1 - \mu)\phi^{\frac{\mu - \rho}{\rho}} + (1 - (1 - \theta)(1 - \mu))\phi^{\frac{\mu + \rho}{\rho}} = 1
\]

- positions of "Height" and "Cavity" (It is sufficient to solve some quadratic equation)

- "break point"

\[
\phi^B = \frac{\rho - \mu}{\rho + \mu} \sqrt{\frac{1 - (2\theta - 1)^2}{\left(\frac{1 + \mu}{1 - \mu}\right)^2 - (2\theta - 1)^2}}
\]

- and one more critical value — "turn point"

\[
\phi^T = \sqrt{\max \left\{ 0, \frac{(1 - \theta)(\theta \rho - (1 + \theta \rho)\mu)}{\theta((1 - \theta)\rho + (1 + \theta \rho)\mu)} \right\}}
\]
Black Hole Condition: \( \mu \geq \rho \)

Contour plot (map) of \( \frac{V_H}{V_F}(\varphi, \lambda) \) under

\( \mu = 0.5, \rho = 0.25, \theta = 0.6 \)

Legend

"Sea": \( V_H < V_F \)
"Shore": \( V_H > V_F \)
"Coastline": \( V_H = V_F \)
“Hooked” Coastline: $\varphi_1^S > \varphi^T$

Narrow Isthmus:
$\varphi_0^S \approx 0.277 < \varphi^U \approx 0.29$
$\mu = 0.25, \rho = 0.75, \theta = 0.5025$

Wide Isthmus:
$\varphi_0^S \approx 0.3 > \varphi^U \approx 0.2535$
$\mu = 0.25, \rho = 0.75, \theta = 0.525$
U-Turn Point $\varphi^U$

$\varphi^U \approx 0.29, \lambda^U \approx 0.7$

$\varphi^U \approx 0.2535, \lambda^U \approx 0.9$
“Regular” coastline $\varphi_1^S \leq \varphi^T$

"Regular" northern coastline:
$\mu = 0.25$, $\rho = 0.75$,
$\theta = 0.560885$,
$\varphi^T = \varphi_1^S \approx 0.216766$

More regular:
$\mu = 0.25$, $\rho = 0.75$, $\theta = 0.6$,
$\varphi^T \approx 0.238 > \varphi_1^S \approx 0.184$
Big (Continental) Asymmetry: \( \theta \geq \frac{\rho + \mu}{2(1-\mu)\rho} \)

\[ \mu = 0.25, \quad \rho = 0.75, \quad \theta = 0.89, \quad \varphi_0 = 1 \]
"Catastrophic" and "Step-by-Step"
Agglomeration/Deindustrialization

"Catastrophe": $\varphi_1^S > \varphi^T$

Step-by-step: $\varphi_1^S \leq \varphi^T$
That’s All, Folks!

Thank You for Attention!
Any questions?