

Financial Markets as Complex Systems

Dr. Amitava Sarkar, Professor
West Bengal University of Technology
Kolkata (Calcutta), INDIA

The study of complex systems in a unified framework has become recognized in recent years as a new scientific discipline, the ultimate of interdisciplinary fields.

- This discipline studies how parts of a system give rise to the collective behaviour of the system, and how the system interacts with its environment.
- The field of complex systems covers different areas of physical and biological sciences, computer science, social sciences and the entire spectrum of management and allied disciplines.
- It focuses on certain questions about parts, wholes and relationships. These questions are relevant to all traditional fields. Because of its interdisciplinary character, there is no single theory of complexity, but a multitude of approaches arising out of the diverse disciplines that span this multifaceted field

- A complex system is to be formally understood as a collection of many interdependent parts that interact with each other through competitive nonlinear collaboration leading to emergent, self-organized behaviour.
- The basic and probably the most significant example of a complex system is intelligent life in general and the human brain in particular. Other examples include governments, families, societies, ecological systems, the weather, economy, organizations, information systems and financial markets.
- It is the last named that will be at the center of our discussion in this paper.
- There are three interrelated approaches to the current study of complex systems:
 - (i) how interactions give rise to patterns of behaviour; (ii) understanding the ways of describing complex systems; (iii) the process of formation of complex systems through pattern formation and evolution.

- It is the study of the emergence of new collective properties qualitatively different from the properties of the “elementary” components of the system that break the traditional boundaries between the different disciplines: the “elementary” objects could belong to one discipline – say chemistry – while the emergent objects could very well belong to another – say biology.
- The methods belong to the “interdisciplinary space”. The main challenge of research into complex systems is exploring and developing this “interdisciplinary” territory .
- The study of complex systems is becoming more and more important because of its ability in understanding indirect effects.

- Certain problems are difficult to comprehend and their solutions hard to obtain because the causes and effects are not obviously related. Since the different parts of a complex system are interdependent, disturbing a complex system often has strange and unexpected consequences. This is abundantly clear when we attempt to solve socio-economic problems or avoid ecological disasters caused by our own actions.
- The field of complex systems provides a number of sophisticated tools, some of them concepts that help us think about these systems, some of them analytical for studying these systems in greater depth, and some of them computer based for describing, modeling or simulating these systems.

- Financial markets can be regarded as model “complex systems”. In fact, they are fascinating examples of “complexity in action”: a real world complex system whose evolution is dictated by the decisions of crowds of traders who are continually trying to win in a vast global “game”.
- Financial markets are dynamical systems which are constantly evolving and which continuously generate huge amounts of data. This dynamic characteristic of a financial market means that time is of crucial importance in the study of financial markets.
- Some of the questions of interest to both practitioners and academicians are the following:
 - How do financial markets behave?
 - Why do financial markets behave in this way?
 - Given the behaviour of the financial markets, what can be done to minimize the risk associated with financial markets?

- In order to answer one or more of the above questions, it is necessary to have an idea about the time evolution (dynamics) of the asset prices. It is, of course, impossible to predict what will happen to a given asset or the market in future.
- The time evolution of financial markets is difficult to describe because of their complex nature. Financial markets provide one of the best documented, and longest running record of a large-scale “complex system”.
- It is not easy to mathematically describe the time evolution of a complex system in a general but useful way. This lacuna motivated us to initiate the present work, where we want to study financial markets, in a very limited way, using the lens of complexity. In particular, we wish to address the following issues related to financial markets:
 - Why are financial markets regarded as complex systems?
 - What are the different levels of complexity present in financial markets?
 - What are the scaling laws operating in financial markets and what are their implications?

- We examine the elements of complexity and the characteristic features of complex systems in Section 2.
- Section 3 discusses why financial markets may be regarded as complex systems and what are the different levels of complexity present in financial time series.
- The scaling properties of financial data including power laws and multi-scaling are investigated in some detail in Section 4.
- Section 5 presents our conclusions and provides an outline of research activity possible in this direction in future.

Simple Systems – are ones in which global properties are inherent in the properties of their component parts. Such systems are additive and scale with increasing numbers of components.

Example – Consider a grain of sand. Mass of a bucket of sand is the sum of the masses of the individual grains.

It is a simple process – additive, linear, completely predictable.

Can be studied top-down or bottom-up by traditional reductionist formalism.

Definition and Examples

Complex systems can be defined by what they are not – *they are not simple ones*. A complex system is to be understood as

- a collection of many interdependent parts
- that interact with each other through competitive nonlinear collaboration
- leading to emergent, self-organized behaviour

The basic and possibly the most significant example of a complex system is intelligent life in general, and the human brain in particular.

Other examples include:

Governments, Families, Societies, Ecosystem, Weather, Economy, **Financial Markets**, Organizations, Information Systems

General features of a Complex System

- Complex systems contain many constituents interacting nonlinearly. The constituents of a complex system are interdependent.
- A complex system possesses a structure spanning several scales.
- A complex system is capable of emergent behaviour.
- The combination of structure and emergence leads to self-organization.
- There is no single point of control in a complex system.
- Many characteristics of a complex system are common across problem domains.

- There is another property of complex systems worth discussing.
- This is a property characteristic of all social systems like ecological groupings, immune systems, social or economic classes, teams, dotcom ventures, nations and supranational corporations.
- In all these organizations, complexity involves interplay between **cooperation and competition**. Examples abound in the fields of economics and social sciences.

There is a special category of complex systems known as Complex Adaptive Systems .

As the name signifies, these systems are capable of changing themselves to adapt to a changing environment. They can also change the environment to suit themselves.

A formal definition runs as follows: A Complex Adaptive System (CAS) denotes an open ended system of many heterogeneous agents which interact nonlinearly over time with each other and their environment and which are capable of adapting their behaviour based on experience.

Open ended means that there is essentially limitless possibility for variability in agent characteristics and behaviour. Typical examples include ant colonies, immune systems, brain, markets and companies.

A financial market generally consists of the so called agents (traders), furnished by varying amounts of capital, & the interaction rules (commercial laws) of the trading platform.

The agents interact among themselves in a highly nonlinear way. Each agent tries to obtain the maximum possible profit, by selling and buying financial assets of all types at different times.

In contrast to such a simple microscopic structure, the macroscopic behaviour of financial markets appears to be surprisingly rich: the seemingly uncorrelated swings of financial indices and the extreme event of a crash constitute typical phenomena of complex systems.

The financial market dynamics is sometimes described using terminology from the theory of turbulence, while the financial crash is compared with a phase transition of a physical many-body system

It is possible to identify indicators for nonlinearity in financial markets, such as speculative bubbles.

One also notes how quickly panic can spread in case of larger losses. These phenomena are examples of typical nonlinear processes called autocatalytic, which are characterized by conditions in which small stimuli can be strongly amplified due to the internal dynamics of the system.

This also indicates the absence of any stable state of equilibrium: if the price of an asset increases, agents will generally tend to buy it, thereby fuelling the increase further; the same holds true in case of falling prices, where again there is not any force pulling down the price to a stable equilibrium value.

Several features of complex systems are quite apparent in the behaviour of financial markets. The emergent behaviour of the market resulting from the interactions of the investors is an example of emergence / aggregation. The individual trading rules and investment rules of thumb can be considered as decision rules in the financial markets.

The disappearance of “anomalies” illustrates the concept of adaptive decision rules.

The nonlinear character of agent interactions is reflected in the fact that cause and effect may not be simplistically linked but may interact to churn out inflated outcomes. An example of a feedback loop in financial markets is provided by the practice of “momentum” investors, who use stock price changes as a buy/sell cue, allowing for self-reinforcing behaviour .

One objection raised against the concept of financial markets as complex systems is that there is no such entity as “the” financial market; rather, there are various markets all over the world, with very different products and even distinct trading laws. Hence, the suggested unified treatment cannot be justified.

Although this is true in principle, there are known phenomena, like fat tails in the distributions of price variations and scale invariances that have been found in the most diverse markets, often with varying parameters characteristic of the particular market, but of the same general form.

This remarkable universality and the existence of typical pattern-like structures are probably due to the common speculative character of all such markets, by which is implied that unique interplay between stochastic external influences and the (deterministic) human psychology, with its strong desire to make profits and fear of losing

The fact that financial markets are continuously monitored means that data exist down to the scale of each single communication of bid and ask of a financial asset (quotes) and at the level of each transaction (trade). Availability of this huge amount of data enables a detailed statistical description of different aspects of the asset prices in a financial market .

Although a complete modeling of the dynamics of financial markets is extremely difficult, the results of these studies indicate the existence of several levels of complexity in the price dynamics of a financial market. In particular, the statistical properties of financial time series are complex with respect to their both temporal and ensemble properties. These different levels of complexity reinforce the idea of a financial market as a complex system possessing a structure spanning several scales .

The first level of complexity is manifest in the statistical properties of a single financial time series. In any financial market, the autocorrelation function of returns monotonically decreases with a very short correlation time. The correlation times can be as short as a few minutes in highly traded stocks . This observation is consistent with the Efficient Market Hypothesis (EMH), the short-range memory between returns being a reflection of the absence of continuous arbitrage opportunities in efficient markets

Nonlinear functions of return like the absolute value or the square of the return are found to be correlated over a time scale much longer than a trading day.

To summarize, the stochastic process underlying the price time series of a financial asset is simultaneously characterized by short range and long range memories and is only asymptotically stationary.

The second level of complexity arises due to the presence of high degree of cross-correlation between the synchronous time evolution of a set of equities.

This is a well known empirical fact observed in financial markets .

For a single trading day, correlation coefficient as high as 0.7 can be observed for some pair of equities belonging to the same economic sector.

The study of cross-correlation of a set of economic entities can improve economic forecasting and modeling of composed financial entities like stock portfolios.

The nature and dynamical properties of the correlations between stocks are thus key aspects of the complexity of a financial market.

The third level of complexity occurring in financial markets arises from the collective behaviour observed during typical and extreme market events. This is the ultimate signature of “emergence” resulting from the interactions among the agents and with the environment.

Typical and extreme days are different with respect to the statistical properties of the ensemble return distribution. It is observed that, in addition to the first moment (mean return), higher order moments governing the shape of the ensemble return distribution change during extreme market events (crashes or rallies).

The change of shape of the ensemble return distribution is, however, not fully arbitrary. Rather, it is possible to detect statistical regularities for extreme events occurring after quite a long time interval. The shape of the ensemble return distribution is found to be symmetrical on the typical trading day while on extreme days (crashes or rallies) the distribution turns out to be skewed (asymmetrical).

One model that is often used to explain the results of the empirical analysis (of collective market behaviour) is the single-index model . This model builds on the assumption that the returns of all assets are controlled by a single factor, viz. the market. For any asset, we have

$$R_i(t) = \alpha_i + \beta_i R_M(t) + \epsilon_i(t), \quad (1)$$

where $R_i(t)$ and $R_M(t)$ are the return of the asset i and of the market on day t respectively; α_i and β_i are two real parameters and $\epsilon_i(t)$ is a zero mean noise term characterized by a variance equal to σ_ϵ^2 . The noise terms of different assets are uncorrelated,

i.e. $\langle \epsilon_i(t) \epsilon_j(t) \rangle = 0$ for $i \neq j$.

The covariance between $R_M(t)$ and $\epsilon_i(t)$ is set equal to zero for any i .

However, comparison of the empirical data with the single-index model shows that correlation detected among the stocks can be described by the single-index model only as a first approximation. The degree of approximation weakens for market days of increasing absolute average return and nearly completely fails to describe the market behaviour for extreme days.

Auto-catalyticity

This is a unifying concept appearing ubiquitously in complex systems.

The dynamics of a quantity is auto-catalytic if the time variations of that quantity are proportional (via stochastic factors) to its current value.

- The “simple” objects (or groups of simple objects) responsible for the emergence of most of the complex collective objects have autocatalytic properties.
- Autocatalyticity ensures that the behaviour of the entire system is dominated by the elements with the highest auto-catalytic growth rate rather than by the typical or average element.

Autocatalytic stochastic growth and power laws

Pareto (1897)

- The wealth of individuals spreads over many orders of magnitude.
- Dynamics of the social wealth is not dominated by the typical individual but by a small class of very rich people.
- Mathematically, it is clear that instead of the usual fixed scale distribution (Gaussian, exponential), the wealth follows a “power law” distribution.
- In spite of the wide fluctuations in the average wealth during different situations, the exponent of the power laws always remains between narrow bounds.
- Similar effects are observed in a very wide range of measurements: meteorite sizes, earthquakes, word frequencies and internet links.

In all these systems, presence of power laws provides a conceptual bridge between the microscopic elementary interactions and the macroscopic emergent properties. The auto-catalytic character of the microscopic interactions governing these systems can explain their behaviour in a unified way.

Introduction to scaling

Scaling laws or power laws (i.e. hyperbolic distributional characteristics of some measurements) are the much sought imprint of complex system behaviour in nature and society .

A power law has the general form

$$Y = Y_0 X^\alpha \quad (2)$$

where Y is some response or dependent variable, X represents an independent or explanatory variable, Y_0 is a normalization constant and α is the scaling exponent.

Equation (2) can be rewritten as

$$Y = f(X), \text{ where } f(X) = Y_0 X^\alpha \quad (3)$$

Power laws are scale-invariant, that is, they display invariance under scale change. This can be seen if we consider a scale transformation in X such that $X \rightarrow \lambda X$, then

$$f(X) = Y_0 X^\alpha \rightarrow Y_0 \lambda^\alpha X^\alpha = \lambda^\alpha f(X) \quad (4)$$

Thus a change in the scale of the independent variable preserves the functional form of the original relationship. Scale invariance describes phenomena that are not associated with a particular or characteristic scale and are also known as scale-free or true on all scales, that is, they possess the same statistical properties at any scale.

For the notion of **scale-invariance** in the context of finance, e.g. on graphs showing the time evolution of stock prices, one always needs to indicate the time horizon they are referring to; if not, it could become impossible to figure out whether what we see shows last week's or last year's price evolution.

A graph with such a property is called self-affine and the underlying process is said to be scale-invariant. With regard to financial markets, scale invariance means that the price evolution process can be described in terms of minutely, hourly, or daily recorded data but the principal property of the process i.e. the distribution of the price variations, will always be of the same general form, with only a scaling parameter that needs to be adjusted for a change of the time scale.

Scaling laws in Finance

The most important observation related to scaling laws in financial markets is the widespread power-law behaviour exhibited by large price changes. This is corroborated for practically all types of financial data and markets. The quantity of interest is the relative price change or return, defined as

$$R_t = (P_t - P_{t-1}) / P_{t-1} \quad (6)$$

where P_t denotes the price of an asset at time t .

Since for daily data, R_t is usually small ($|R_t| \leq 0.2$), one often uses the log-differences (continuously compounded returns),

$$R_t \cong \ln(P_t) - \ln(P_{t-1}) \quad (7)$$

Statistical analysis of daily returns indicates strong evidence for hyperbolic behaviour of the tails of the distribution, viz.

$$\text{Prob}(|R_t| > X) \cong X^{-\alpha} \quad (8)$$

The tails of a distribution denote the regions far from the centre. Reasonable definitions of the “tails” would be that the left tail is the region from $-\infty$ to $(\mu - 2\sigma)$ and the right tail is the region from $(\mu + 2\sigma)$ to ∞ although the choices of $(\mu - 2\sigma)$ and $(\mu + 2\sigma)$ are somewhat arbitrary.

Scaling Laws in Finance

The most important observation related to scaling laws in financial markets is the widespread power-law behaviour exhibited by large price changes. This is corroborated for practically all types of financial data and markets.

The discovery of a power law is interesting because it points to the existence of a truly universal property in the socio-economic milieu. This law has a rigorous statistical basis, similar to that existing for similar universal constants occurring in the natural sciences: financial markets continuously generate large amounts of data at all frequencies and power-law scaling is confirmed for this huge data set without exception.

Significant amount of work has been carried out in relation to the above scaling law. Initial work in this direction was due to Mandelbrot and Fama who reported leptokurtosis in cotton futures and proposed the Levy distribution as a statistical model for asset returns. Subsequent studies were conducted, among others, by Jansen & de Vries, Lux etc. Stanley et.al have reported a number of studies on the probability distributions of stock price variations, one of the prominent of these being the “universal cubic law” of asset returns.

Scaling Laws in Finance

Very strong correlation of volatility over time is also observed. Greater the volatility in today's market, higher would be the fluctuations of market prices in the next periods. Typical flip-flops are observed between tranquil and more turbulent episodes in the data (“volatility clustering”). This dependency can be useful for predicting the future development of volatility – an important task in risk and portfolio management.

Long term dependence was first shown by Ding, Engle & Granger in the Economics literature and also by Liu et.al and Vandewalle & Ausloos in the Physics area. In the econophysics literature, measurement of long range dependence is mostly based on estimation of the Hurst exponent from Mandelbrot's R/S analysis or the refined detrended fluctuation analysis. Efforts have also been made to transcend the original GARCH model and adopt FIGARCH and other long-memory stochastic volatility models that allow for hyperbolically decaying autocorrelations.

There are studies that have reported additional power laws for high-frequency data from the US stock market [50]:

- (a) the unconditional distribution of volume in this data is found to follow a scaling law with exponent ~ 1.5
- (b) the number of trades per unit time is claimed to follow a power law with index ~ 3.4 .

The Multi-Scaling Approach

Multi-scaling refers to processes or data which are characterized by different scaling laws for different moments. These defining features can be captured by dependency of the temporal scaling parameter on the relevant moment.

Vasilicos et.al[53] first demonstrated multi-scaling properties of financial data in an empirical paper. Similar studies were also done by, among others, Ausloos & Vandewalle, Ghasghaie et.al etc.

The multi-fractal framework was extended further by Mandelbrot and others. They proposed a compound stochastic process in which a multi-fractal cascade is used for time transformations. It was assumed that the price returns $R(t)$ were described as

$$R(t) = B_H [\theta(t)]$$

where $B_H []$ is the fractional Brownian motion with index H and $\theta(t)$ is a distribution function of multi-fractal measure.

The function $\theta(t)$ has a sense of “trading time” that reflects intensity of the trading process. Research indicates that both the binomial cascade and the lognormal cascade embedded into the Wiener process (i.e. into $B_H []$ with $H = 0.5$) may yield a more accurate description of several financial time series than the GARCH model. This is an active frontier of research, and new findings in empirical research will continue to pose new challenges for theoreticians

The Quantitative Characterization – Stylized facts I

Stylized Facts - empirical observations that are so consistent and have been made in so many contexts (e.g., across a wide range of instruments, markets and time periods) that they are accepted as truths, to which theories must fit.

They are universal regularities, independent of time, place and specific compositional details.

Obtained by taking a common denominator among the properties observed in studies of different markets and instruments.

Some of these are :

- Absence of autocorrelation of asset returns
- Slow decay of autocorrelation in absolute returns – sign of long-range dependence
- Fat tails – return distribution appears to display a power-law or Pareto-like tail with a tail index which is finite and greater than two (positive excess kurtosis)

The Quantitative Characterization – Stylized facts II

- Excess Volatility – not easy to justify the observed level of volatility in asset returns by taking into account variations in the fundamental economic variables. Large (positive or negative) returns cannot always be explained by the arrival of new information in the market.
- Volatility Clustering – different measures of volatility display a positive autocorrelation over several days, which means that high and low volatility events tend to cluster in time; periods of intense and mild fluctuations tend to cluster together.
- Volatility persistence (long memory) – dependence between stock market returns at different times; volatility has slowly decaying autocorrelations and there is nonlinear dependence.

Implications –

Normal distribution inadequate to describe the return distribution; necessary to invoke power-law type distribution (signature of complexity)

Common underlying mechanism responsible for these facts.

Stylized facts may be interpreted as emergent properties of complex system – the financial market.

The Road Ahead

This paper has discussed why financial markets may be regarded as model “complex” systems by mapping the distinctive features of financial markets to those of a candidate complex system.

However, one important feature might be incorporated while constructing such a model. It is necessary to understand that a system turns out to be complex because the structure embedded in such a system out competes the more simple structures. The evolution of a system into a complex form becomes possible only if such evolution makes the system robust and fit. Thus any model constructed should be able to display emergent complexity.

Signatures of complexity should emerge from the dynamics of these models because they would be more robust and fit than the seemingly simple systems. Thus a complete model of a financial market should be characterized by both complexity and robustness and be able to identify the circumstances in which one reinforces the other.

This needs to be done at both phenomenological and theoretical levels. Studies in such directions with greater quantitative emphasis will be the subject of our investigation in future.

Thank You